Quiz 05

Dec 22, 2017.

1. Use any desingularization method to evaluate the improper integral

$$\int_0^1 x^{\frac{-1}{3}} e^x \, dx$$

with a composite quadrature rule of 4th order accuracy. Demonstrate numerically that the result is indeed 4th order accurate.

2. Let A be an $N^2 \times N^2$ matrix with $a_{ij} = 0$ except for $-N \le i - j \le 2N$ (only $3N + 1$ diagonals have nonzero entries). Suppose that $Ax = b$ can be solved using Gaussian elimination without pivoting. Find the leading order operation count (multiplication/division only) CN^p for the elimination and backward substitution, respectively. Give all details.

3. Write a pseudo-code for Gaussian elimination, assuming no pivoting is needed. Then another pseudo-code for backward substitution. Partial credits is nearly impossible for incorrect algorithms. Check carefully.

4. Perform the required row interchanges for the following linear system using scaled partial pivoting:

$$\begin{aligned} & x_2 + & x_3 &= 6 \\ x_1 - & 2x_2 - & x_3 &= 4 \\ x_1 - & x_2 + & x_3 &= 5 \end{aligned} , \tag{1}$$

Give all details and the final linear system. Need not solve it.

5. Find P, L, U in the factorization $PA = LU$ for the matrix

$$A = \begin{pmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{pmatrix} \tag{2}$$

where P is the permutation matrix corresponding to partial pivoting.