

Quiz 04

Nov 24, 2017.

1. Give an approximation, $f_h''(x)$, of $f''(x)$ from $f(x-h)$, $f(x)$ and $f(x+h)$. Then derive an error identity of the form $f''(x) - f_h''(x) = C_1 f^{(?)}(\xi)h^?$.
2. Find $\min_{h>0} e(h) = \min_{h>0} |f''(x) - f_h''(x)|$ where $f_h''(x)$ is the three point formula from previous problem. Express the critical value h^* and the minimum $e(h^*)$ in terms of machine ε as $O(\varepsilon^\alpha)$ and find α for them.
3. Derive a fourth order approximation of $f'(x)$ from $f(x)$, $f(x \pm h)$, $f(x \pm 2h)$, $f(x \pm 3h), \dots$. Assume $f \in C^\infty$ and show that your formula satisfies $|f'(x) - f_h'(x)| \leq Ch^4$.
4. Let $x_0 = a$, $x_1 = \frac{a+b}{2}$, and $x_2 = b$. Write down trapezoidal rule, the midpoint rule and Simpson's rule approximations of $\int_a^b f(x)dx$. Then derive the error formula (equality) for any one of them of your choice.
5. Evaluate the degree of precision for all three methods in previous problem. Give definition and show details. You may choose any a, b for your convenience ($-1, 1$ are recommended).

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