Quiz 03

Nov 10, 2017.

1. Let P_n be the degree n interpolating polynomial of $\cos(2x)$ on the uniformly spaced nodes $x_0, \cdots, x_n$ on $[0,1]$ with $x_j = jh$, $h = 1/n$. Is it true that

$$\max_{0\le x\le 1} |\cos(2x) - P_n(x)| \to 0 \quad \text{as } n \to \infty?$$

   Explain.

2. Denote by $P_{0,1,\cdots,k}(x)$ the Lagrange interpolating polynomial on the data set $(x_0, f(x_0)), (x_1, f(x_1), \cdots (x_k, f(x_k))$. Express $P_{0,1,\cdots,k}$ in terms of $P_{0,1,\cdots,j-1,j+1,\cdots,k}$ and $P_{0,1,\cdots,i-1,i+1,\cdots,k}$. Then verify your answer indeed is the Lagrange interpolating polynomial.

3. A natural cubic spline S on $[0,2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & 0 \le x \le 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & 1 \le x \le 2 \end{cases}$$

   Find b, c, d.

4. Suppose that we are to construct a piecewise polynomial interpolation $S(x)$ on the data $(x_0, f(x_0)), (x_1, f(x_1)), \cdots, (x_n, f(x_n))$, with additional continuity conditions for S', S'' and S''' on the interior nodes $x_1, \cdots, x_{n-1}$. If we use polynomials of the same degree on each of the interval $[x_0, x_1], \cdots, [x_{n-1}, x_n]$, what is the minimal degree needed in each interval? How many additional end conditions are needed? Count carefully and explain.

5. Given four data $(x_i, \exp(-2x_i))$: $(0.3, 0.5488)$, $(0.4, 0.4493)$, $(0.5, 0.3679)$ and $(0.6, 0.3012)$ (you should generate the data yourself to avoid typo in inputting data). Use Inverse Interpolation to find the root of $x = \exp(-2x)$. You can use any algorithm for Lagrange interpolation. Extra credits for Neville's method (or divided difference, if anyone knows it). After finding x, check yourself that $x = \exp(-2x)$ in indeed satisfied in case of a bug in your code. Need not show the last part.

   Hand in code, put all data within the code so that it can be executed immediately.

Name your codes in the same format as $s104000001.m$ or $s103000002.c$ and make sure it is executable/compilable.