Quiz 03

Nov 14, 2014.

1. Give (need not derive) an approximation of $f''(x)$ from $f(x-h)$, $f(x)$ and $f(x+h)$. Then derive an error identity of the form $f''(x) - f_h''(x) = C_1 f^{(n)}(\xi) h^{n+k}$.

2. Derive a fourth order approximation of $f'(x)$ from $f(x)$, $f(x \pm h)$, $f(x \pm 2h)$, $f(x \pm 3h), \cdots$.

3. At least how many points among $f(x)$, $f(x \pm h)$, $f(x \pm 2h)$, $f(x \pm 3h), \cdots$ are needed to approximate $f^{(4)}(x)$? What is the order of the truncation error p in $f^{(4)}(x) - f_h^{(4)}(x) = O(h^p)$? Explain. You could give your answer with or without explicitly finding the coefficients of $f(x)$, $f(x \pm h)$, $f(x \pm 2h)$, $f(x \pm 3h), \cdots$.

4. Suppose that $M = N_1(h) + K_1 h^2 + K_2 h^4 + + K_3 h^6 + \cdots$ and $N_1(h)$, $N_1(h/2)$, $N_1(h/4)$ are given. Construct $N_3(h)$ using an extrapolation table or line by line derivation. What is the order of accuracy for $N_3(h)$?

Numerical Analysis I, Fall 2014 (http://www.math.nthu.edu.tw/~wangwc/)

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