

Quiz 02

Oct 17, 2014.

1. The sequence $a_n = 1 + \frac{1}{n^2}$ converges to its limit of order α . Find α . Then give an example of a quadratically convergent sequence.
2. True or False? Prove it if true, give a counter example if false.
If $x = g(x)$ has exactly one root x^* in $[a, b]$, $g([a, b]) \subseteq [a, b]$ and $\underline{|g'(x)| \leq 1}$ on $[a, b]$, then the standard fixed point iteration converges to x^* for any $x_0 \in [a, b]$. Note that the underlined condition is not standard.
3. Find a root of $x = 2 \cos x$ with 10 correct decimal digits using Newton's method. Put (1): the iteration formula, (2): x_0 and (3): the answer x^* , on the answer sheet, but do not hand in the code. You may use the code in the package or write the code yourself.
4. Show that, if x^* is the unique root of f , $f \in C^2[x^* - \delta, x^* + \delta]$, $f' \neq 0$ on $[x^* - \delta, x^* + \delta]$ and $f([x^* - \delta, x^* + \delta]) \subseteq [x^* - \delta, x^* + \delta]$. Then Newton's method satisfies $|x_{n+1} - x^*| \leq C|x_n - x^*|^2$.
5. The first few iteration $(p_i, f(p_i))$, $i = 0, 1, 2, 3$ of method of false position for some equation $f(x) = 0$ is given in q2p5.txt. Find p_4 . Also give your formula for finding p_4 and explain.

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