Quiz 04

Nov 26, 2010.

1. (25 pts) Let $x_0 = -1, x_1 = 0, x_2 = 1$. To interpolate f and f' on x_0, x_1, x_2 with a single polynomial, what is the lowest degree needed in general? Find a polynomial $P(x)$ of this degree such that $P(x_0) = P'(x_0) = 0$, $P(x_2) = P'(x_2) = 0$ and $P(x_1) = 1$, $P'(x_1) = 2$ (Need not simplify).

2. (25 pts) Write a pseudo-code (algorithm) for the divided difference version of Hermite interpolation with data $(x_0, f(x_0), f'(x_0)), \cdots (x_n, f(x_n), f'(x_n))$. You can instead give (or start with) the original version (using the 'basis functions') for 15 pts.

3. (25 pts) A clamped cubic spline S for a function f is defined by
$$S(x) = \begin{cases} S_0(x) = 1 + Bx + 2x^2 - 2x^3, & 0 \le x \le 1 \\ S_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3, & 1 \le x \le 2 \end{cases}$$
Find $f'(0)$ and $f'(2)$.

4. (25 pts) Interpolate the function $f(x) = \frac{1}{1+x^2}$ on $[-5, 5]$ with equally spaced nodes $-5, -4, \cdots, 4, 5$ using the built-in spline function in octave. Give the interpolated value at $x = 0.1$ with 10 digits. The answer should be close to $f(0.1)$. You can write your code to the answer sheet for potential partial credit in case the answer is wrong.

Numerical Analysis I, Fall 2010 (http://www.math.nthu.edu.tw/~wangwc/)

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