Final Exam

Jan 14, 2011. Show all details. 16 points for problem 4, 14 points each for other problems.

1. Integrate $\int_0^1 \left(\sin x - 0.5(\cos 1 - 1)x^2\right) dx$ using midpoint Rule. The exact answer is $\frac{7}{6}(1 - \cos 1)$.
   (a) Estimate theoretically n or h it takes to bound the error within 10^{-6}.
   (b) Find the order of convergence numerically. You can use the exact answer (or not).
   (c) (Harder, extra point) Why is the numerical order greater than 2?

2. Use local Taylor's expansion to show that the error for Simpson's rule satisfies
$$\left| \int_{-1}^{1} f(x)dx - (af(-1) + bf(0) + cf(1)) \right| \le C \max_{\xi \in [-1,1]} |f^{(4)}(\xi)|$$
provided $f \in C^4[-1, 1]$. The constants a, b, c are not given, but you should know them. You need NOT find the best constant C and need NOT show that equality holds.

3. Derive a quadrature of the form
$$\int_{-1}^{1} f(x) = a(f(c) + f(-c)) + bf(0)$$
with largest degree of precision, where $a, b \in R$ and $0 < c < 1$ are to be determined. Derive the equations for a, b, c and need not solve them. What is the actual smallest integer p such that $\int_{-1}^{1} x^p dx$ is NOT exact for this quadrature rule?

4. Use any method to find the area enclosed by $x^4 + y^4 = 1$ to 10 digits. You need to explain why your answer is correct to 10 digits, either theoretically or numerically. Extra credits will be given for efficient methods.

5. Show that the formula $Q(P) = \sum_{i=1}^{n} c_i P(x_i)$ can not have degree of precision greater than $2n - 1$, regardless of the choices of $c_1, \cdots, c_n$ and $x_1, \cdots, x_n$.

6. Find the row interchanges that are required for solving
$$\begin{array}{rrrl} x_1 & -5x_2+ & x_3 & = 7 \\ 10x_1 & + & 20x_3 & = 6 \\ 5x_1 & - & x_3 & = 4 \end{array} \tag{1}$$
by Gaussian elimination with (a) partial pivoting, and (b) scaled partial pivoting, respectively.

7. Write a pseudo-code for LU decomposition with $l_{ii} = 1$. You can use any version, direct or recursive. Derive it first. You may assume pivoting is not needed and need not check for pivoting in your pseudo-code (which is not the case in reality, of course).