

Midterm 02

Dec 04, 2009.

Attach relevant functions at the end of the main program and name it u916xxxx_pr8.m.

1. Give the error formula of polynomial interpolation of x^{n+1} on $0 = x_0 < x_1, \dots < x_n = 1$.
1. What is the smallest possible error $\max_{[0,1]} |x^{n+1} - p_n(x)|$ by varying the nodes $0 = x_0 < x_1, \dots < x_n = 1$?
2. Given the knots x_0, \dots, x_n , What would be the matching conditions on x_1, \dots, x_{n-1} for a piecewise quadratic spline? Propose a proper boundary condition to uniquely determine the spline functions.
3. Find the Chebyshev polynomials of degree n and corresponding Chebyshev nodes on $[0, 2]$.
- 4.
- 5.
6. Find the least square approximation of x^3 on $\text{span}\{1, x, x^2\}$ on the interval $[0, 1]$.
7. Derive the error formula for the Trapezoidal rule.
8. Derive the Richardson extrapolation of the trapezoidal rule.
9. Write down the equations satisfied by the nodes and weights for the weighted Gaussian Quadrature Formulas for $\int_0^1 \sqrt{x} f(x) dx$ with $n = 2$ and solve for the nodes and weights.
10. Given $x_0 = 0, x_j = jh, \dots, x_N = 1$ where $h = 1/N$ and $f(x_j), j = 0, 1, \dots, N$. Give an $O(h^2)$ approximation of $f'(0)$ and $f''(0)$. using method of undetermined coefficients.