Numerical Analysis I, Fall 2009 (http://www.math.nthu.edu.tw/~wangwc/)

## Quiz 06

Jan 05, 2010.

- 1. Let A, B be two real  $n \times n$  matrices. Show that  $||AB|| \leq ||A|| ||B||$ . Here the matrix norm is not specified (it could be generated from  $L^2$ ,  $L^{\infty}$  or other norms in  $\mathbb{R}^n$ ). You can prove it for one particular norm, or give a proof that works for any norm.
- 2. Show that  $\operatorname{cond}(A) \ge 1$  for any A.
- 3. Describe Jacobi iteration and Gauss-Siedel iteration.
- 4. Let  $A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ -1 & 1 & 3 \end{pmatrix}$ . Give a convergent iteration method for solving Ax = b. Explain why it converges

5. Let 
$$B = \begin{pmatrix} 0.5 & 1 & 0 \\ 1 & 0.5 & 0 \\ -1 & 1 & 0.5 \end{pmatrix}$$
. Give a convergent iteration method for solving  $Bx = b$ .  
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