Quiz 04

Nov 27, 2009.

1. Find an orthonormal (or orthogonal) basis for Span$\{1, x, x^2\}$ on $[-1, 1]$ with standard inner product (If you happen to know the answer from memory, that will do. Just verify it). Then find the least square quadratic approximation of e^x on $[-1, 1]$.

2. Find a weight function $w(x)$ such that the Chebyshev polynomials $\{T_k(x), k \geq 0\}$ on $[-1, 1]$ satisfy $(T_i, T_j)_w = 0$, $i \neq j$. Explain.

3. Write down the formula for Trapezoidal rule and Simpson rule for approximating $\int_0^1 f(x)dx$ with 100 uniformly spaced partitions $x_0 = 0, x_1, \cdots, x_n = 1$.

4. The numerical integration for some quadrature rule computed using 100, 200 and 400 uniformly spaced partitions are 1.4663, 1.5861 and 1.6160 respectively. What do you think is the order of accuracy of this quadrature rule?

5. Write down the equations satisfied by the nodes and wrights of Gaussian Quadrature Formulas with $n = 3$ on $[-1, 1]$. Need NOT solve for the nodes and weights.

Numerical Analysis I, Fall 2009 (http://www.math.nthu.edu.tw/˜ wangwc/)

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