Quiz 03

Nov 13, 2009.

1. Find the Newton divided difference $f[0,1,2,3,4]$ where $f(x) = x^4 + \pi x^3 - x^2 + 1$.
2. $$s(x) = \begin{cases} x^3 + 2x^2 - x + 1 & -1 \le x \le 0 \\ \alpha x^3 + \beta x^2 + \gamma x + \delta & 0 \le x \le 1 \end{cases}$$
   Find ALL values of $(\alpha, \beta, \gamma, \delta)$ such that $s(x)$ is a cubic spline on $[-1,1]$.
3. Define the Chebyshev polynomial $T_n(x)$ on $[-1,1]$ and find its roots.
4. State (need not prove) the error formula for polynomial interpolation. Give an error bound in sup-norm for polynomial interpolation of e^x on $[-1,1]$ using roots of $T_5(x)$ as interpolating points.
5. Does
   $$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \tan^3(\frac{i\pi}{2n})$$
   converge? If so, what is the limit? If not, what is the leading order term in terms of n?

Numerical Analysis I, Fall 2009 (http://www.math.nthu.edu.tw/~wangwc/)

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