Midterm 02

Dec 04, 2009.

1. Give the error formula of polynomial interpolation of the function $f(x) = x^{n+1}$ on $0 = x_0 < x_1, \cdots < x_n = 1$. What is the smallest possible error $\max_{x\in[0,1]} |x^{n+1} - p_n(x)|$ by varying the nodes $0 = x_0 < x_1, \cdots < x_n = 1$?

2. Given the knots $x_0, \cdots, x_n$, what would be the matching conditions on $x_1, \cdots, x_{n-1}$ for a piecewise quadratic spline? How many boundary conditions are needed to uniquely determine the spline function? Explain.

3. Does
$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \sec(\frac{i\pi}{2n})$$
converge? If so, find the limit. If not, find the leading term.

4. Find the least square approximation of x^3 on span$\{1, x, x^2\}$ over the interval $[0, 1]$.

5. Derive the quadrature rule obtained from applying Richardson extrapolation to the Trapezoidal rule.

6. Write down the equations satisfied by the nodes and weights for the weighted Gaussian quadrature formulas for $\int_0^1 \sqrt{x} f(x)\, dx$ with $n = 2$. Need NOT solve for the nodes and weights.

7. Given $f(x_j)$, $j = 0, 1, \cdots, N$ and $x_0 = 0$, $x_j = jh, \cdots, x_N = 1$, where $h = 1/N$. Write down an $O(h^2)$ approximation of $f'(0)$ and $f''(0)$ using method of undetermined coefficients.

8. (Programming)

   Use Simpson rule to find the numerical value of $\int_0^1 \frac{1}{1+\sin(x)}\, dx$. Use Richardson extrapolation to analyze your numerical results and find out how many grid points are needed to give 7 correct digits. Then write down 7 correct digits in the answer sheet as well. Attach relevant functions at the end of the main program and name it u916xxxx_pr8.m.

9. (Programming)

   Interpolate $g(x) = \frac{1}{1+(x-5)^2}$ on $[0, 5]$ with the nodes $x_j = 5(\frac{j}{N})^p$, $j = 0, 1, \cdots, N$. Then report the maximum error on $0 : 0.001 : 5$. Do this for $N = 10$ and $N = 20$ and $p = 1, 2$ respectively. Write down these four errors on the answer sheet. Attach relevant functions at the end of the main program and name it u916xxxx_pr9.m.