Midterm 1

Oct 27, 2009.

Downlowd relevant program from http://www.math.uiowa.edu/ftp/atkinson/ENA_Materials.

1. How many terms of Taylor polynomial are needed to approximate e^x within 10^{-6} on $[-1,1]$? Explain.

2. Write down Taylor's polynomial $p_3(x,y)$ for $f(x,y)$ around the origin $(0,0)$, assuming $f \in C^\infty(R^2)$. That is, $f(x,y) = p_3(x,y) + R_3(x,y)$, degree of $p_3 = 3$. Write down p_3 (need NOT give R_3).

3. Solve for $x^2 - 1900x + 1 = 0$ to 15 correct digits. Explain your method, implement it in matlab and then copy your computed result carefully. Every digit counts.

4. How many "bits" does it take to store a floating point number in the range

$$\pm 1.d_1 d_2 \cdots d_s \times 2^e$$

with $s = 23$, $d_j \in \{0,1\}$, $-126 \le e \le 127$? What is the largest possible rounding error caused by floating number representation in this case? In other words, what it the largest possible number of $|\frac{x - fl(x)}{x}|$, where x is a real number lying in the representable range specified above and $fl(x)$ is the floating number representation of x obtained by "rounding"? Explain.

5. Suppose that a floating point algorithm gives rises to $|\frac{x - fl(x)}{x}| \le \epsilon$, give an estimate of the relative error of evaluating 2^{xy} with $x = 0.9$, $y = 1.9$.

6. Give the formula of Newton's method for solving $x^2 - 2x + 1 = 0$. Find the order of convergence and prove your answer.

7. Given the data set $\{x_0, y_0\} = (0,2)$, $\{x_1, y_1\} = (1,1)$, $\{x_2, y_2\} = (2,3)$, $\{x_3, y_3\} = (3,0)$, Write down the Lagrangian polynomials $L_i(x)$, $i = 0,1,2,3$ and the interpolating polynomial in terms of L_i and y_i. Need NOT simplify. Then prove that $L_0(x) + L_1(x) + L_2(x) + L_3(x)$ is a constant function.

8. (Analysis and Programming)

   Prepare a matlab program for evaluating

$$g(x) = \int_0^x e^{-t^4} dt$$

   Find its Taylor polynomial so that the error is bounded by 10^{-9} for $|x| \le 1$. Let your main program plot the graph of $g(x)$ on $[-1,1]$ and show $g(1/2)$ on screen. Attach relevant functions at the end of the main program and name it u916xxxx_pr8.m.

9. (Analysis and Programming) Use fixed point iteration to find the solution to

$$x = -2\sin x + 0.1$$

   If your fixed point method does not converge, you can use Newton's method (half credit) or bisection (1/4 credit). Attach relevant functions at the end of the main program and name it u916xxxx_pr9.m.

   Create a directory on your desktop called u916xxxx and put your source code in it.