

Preparation guide for midterm 02

Dec 01, 2009.

Most of the exam problems will be closely related to your homework problems. Check the list below and then if you are not confident with some of the items, study the textbook and related homework problems.

1. Memorize the error formula of general polynomial interpolation. Be able to derive the error bound for some particular polynomial approximation such as the near-minimax approximation.
2. Understand the meaning of spline. Understand why cubic polynomial matches up to second derivative at knots. Understand how to solve for the spline polynomials with various boundary conditions such as 'natural', 'clamped' and 'not-a-knot'.
3. Memorize definition of Chebyshev polynomials and Chebyshev nodes. Pay attention on intervals which are not on $[-1, 1]$.
4. In problems like

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \tan^3\left(\frac{i\pi}{2n}\right)$$

Be able to read out the leading order term of the integrand and the integral.

5. Understand properties of Legendre polynomials and its relation with Gram-Schmidt procedure. Again, pay attention on intervals which are not on $[-1, 1]$.
6. Be able to derive equations for general (weighted) least square approximations, either with or without an orthogonal basis.
7. Study the quadrature and error formula for Midpoint, Trapezoidal and Simpson rules.
8. Understand and be able to apply the Richardson extrapolation.
9. Understand the principle and derivation of the (weighted) Gaussian Quadrature Formulas and the error bound.
10. Understand the principle of numerical differentiation. Be able to derive formula using undetermined coefficients.