Final Exam

Dec 15, 2009. 10 points each for problems 1-6. 20 points each for problem 7 and 8.

1. Find the matrix representation of reflecting across a plane whose normal is $(1,1,1)/\sqrt{3}$.

2. Find inverse of
$$A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \tag{1}$$
by means of Gaussian elimination.

3. Find the Choleski decomposition ($A = LL^T$) of the matrix given in (1).

4. Show that the iteration $Nx^{(k+1)} = b + Px^{(k)}$ converges if $M = N^{-1}P$ satisfies $\|M\| < 1$ for some matrix norm $\|\cdot\|$.

5. Is the condition $\|M\| < 1$ necessary for convergence in previous problem? If so, prove it. If not, give a counter example.

6. Write down and derive the formula for condition number.

7. (Programming)

   Solve for $Bx = b$ with an iterative method, where
$$B = \begin{pmatrix} 1/2 & -1 & 0 \\ 1 & 1/2 & 0 \\ -1 & 1 & 1/2 \end{pmatrix} \tag{2}$$
   and $b = (1,1,1)^T$. To get a convergent iteration, try with a proper splitting $B = N - P$ and analyze the eigenvalues of the corresponding M. If you don't know how to analyze it, you can use matlab to help you find the eigenvalues of M.

   If you cannot find a convergent iteration, change the diagonal entries to -3. If you do this one correctly, you get (a little) partial credit. Attach relevant functions at the end of the main program and name it u916xxxx_pr07.m.

8. (Programming)

   Use the power method (only) to find the largest and smallest (both in absolute value) eigenvalue of
$$C = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \tag{3}$$
   You can use the matlab built-in function to check your answer, but you have to obtain them by power method to get points for this problem. Attach relevant functions at the end of the main program and name it u916xxxx_pr08.m.