Midterm 01

Oct 31, 2014. Hand in your code for problem 1, 2, 6 for possible partial credit in case your numerical answer in wrong.

1. (20 pts) Use any method to find a solution of $(1+x)^{1/3} - (1-x)^{1/3} = 10^{-10}$ to 10 correct digits.

2. (12 pts) Find p_{20} to 10 correct digits if $p_0 = 1$, $p_1 = 0.9$, $p_n = 4p_{n-1} - 2.79p_{n-2}$.

3. (10+6 pts)

   (a) Give a locally convergent fixed point iteration for solving $f(x) = x - 3\sin(x) - 0.01$. Just analyze and give the formulae, need not find the numerical solution.

   (b) Give an upper bound for the number of steps it takes to reach $|x_n - x^*| < 10^{-6}$ with $x_0 = 1$.

4. (12 pts) Derive Aitken's Δ^2 method from the assumption

$$\frac{p_{n+1} - p}{p_n - p} \approx \frac{p_{n+2} - p}{p_{n+1} - p}$$

5. (12+12 pts)

   (a) Give a cubically convergent method to solve for $e^x - 1 = 0$. Give the formula and prove that it is cubically convergent (locally). If you cannot do it, do the same for a quadratically convergent method for partial credit.

   (b) Find α and λ (the constants in the definition of order of convergence) analytically for your method if it is applied to solve the equation $e^x - x - 1 = 0$ instead. If cannot do it, do the same for a quadratically convergent method for partial credit (no matter whether you did cubically or quadratically convergent method in (a)).

6. (20 pts) Find $P(0.3)$ where $P(x)$ is the Lagrange polynomial interpolating the data $(0, \sin 0), (0.25, \sin 0.25), (0.5, \sin 0.5), (0.75, \sin 0.75), (1, \sin 1)$. Give formula and find the numerical value.

7. (12 pts) Suppose that we are to construct a piecewise polynomial interpolation $S(x)$ on the data $(x_0, f(x_0)), (x_1, f(x_1)), \cdots, (x_n, f(x_n))$, with additional continuity conditions for S', S', S'' and S''' on the interior nodes $x_1, \cdots, x_{n-1}$. If we use polynomials of the same degrees on each of the interval $[x_0, x+1], \cdots, [x_{n-1}, x+n]$, what is the minimal degree needed in each interval? How many additional end conditions are needed? Explain.