Study guide for Chap 04, Numerical Differentiation and Integration

1. Coverage of this study guide: 4.1: Numerical Differentiation (11), 4.2: Richardson Extrapolation (8), 4.3: Elements of Numerical Integration (10), 4.4: Composite Numerical Integration (10), 4.7: Gaussian Quadrature (7), 4.9: Improper Integrals (5).

2. Learn to derive formula for 2nd order and higher order numerical differentiations and their error estimates (and error identities, if possible). The centered differences are basics. Also derive the formula on end points and non-uniform spacing.

3. Study the effect of round-off instability on numerical differentiations including different orders of derivatives ($f^{(n)}(x)$ with various n) and different orders of accuracy ($O(h^p)$ with various p).

4. Study how to use Richardson's extrapolation to obtain more accurate approximations including numerical differentiation and numerical integration. Also study how to obtain numerical order of accuracy when the true solution is not known.

5. Implementation of elementary numerical quadratures such as composite trapezoidal rule, composite midpoint rule and composite Simpson's method.

6. Study the error bound of above mentioned elementary numerical quadratures, both in terms of the Intermediate Value Theorem and in terms of the integral of higher order derivatives as shown in exercise set 4.4.

7. Study the definition of degree of precision, and how to derive the (closed and open) Newton-Cotes formula from degree of precision.

8. Study the relation between of degree of precision and the error formula. (That is, if the degree of precision in n, what would be the general form of the error of the numerical quadrature?)

9. Study Gaussian quadrature on how to obtain the quadrature points and the weights. Both from the degree of precision approach and the roots of Legendre polynomials' approach.

10. Study how to treat the singular parts of improper integrals. Both on the unbounded integrand case and the infinite domain case.