# Preparation guide for Quiz 04

The exam problems will be closely related to your homework problems. Make sure you understand all of them.

1. Memorize 3-point midpoint formula (i.e. formula involving $f(x_0)$ and $f(x_0 \pm h)$) for $f'(x_0)$, $f''(x_0)$ and study the derivation of their error formula (equality).

2. Study the derivation of 3-point endpoint formula. That is, formula involving $f(x_0)$, $f(x_0+h)$ and $f(x_0+2h)$ or formula involving $f(x_0)$, $f(x_0-h)$ and $f(x_0-2h)$, for $f'(x_0)$ and $f''(x_0)$. You can either use the method of undetermined coefficients (recommended) or the error formula for Lagrange interpolation as done in section 4.1, page 174-175 of the textbook. Another recommended derivation is to use 2-point endpoint formula together with Richardson extrapolation.

3. Study the derivation of 5-point midpoint formula for $f'(x_0)$, $f''(x_0)$, $f'''(x_0)$ and $f^{(4)}(x_0)$ using the recommended methods mentioned above. Pay attention on the relation between the number of data points (five) and the resulting order of accuracy.

4. Study the derivation of 5-point midpoint formula for both the case involving $f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$, $f(x_0+3h)$ and $f(x_0+4h)$, and the case involving $f(x_0-h)$, $f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$ and $f(x_0+3h)$.

5. Study the round-off error instability, first on the text book case (2nd order approximation of $f'(x_0)$), then the general case of $p$th order approximation of $f^{(q)}(x_0)$.

6. Study the procedure of Richardson extrapolation and apply it to 3-point midpoint formula of $f'(x_0)$ and $f''(x_0)$ to get 5-point midpoint formula for $f'(x_0)$ and $f''(x_0)$. Note that, instead of the textbook procedure $h \to \frac{h}{2}$, it is more convenient derive it with the procedure $h \to 2h$.

7. Memorize the trapezoidal rule, the midpoint rule and Simpson's rule. Then study the derivation of their error formula. For Simpson's rule, you only need to go through up to the point when error term can be written as $-\frac{h^5}{12}\left(\frac{1}{3}f^{(4)}(\xi_2)-\frac{1}{5}f^{(4)}(\xi_1)\right)$.

8. Review the definition and procedure to check degree of precision of a given quadrature rule.