HW14

HW #1. §6.2

The details are left as exercises.

- #1a. none
- #3a. Interchange rows 1 and 2.
- #5a. Interchange rows 1 and 3, then interchange rows 2 and 3.

HW #2. §6.5 #4ac.

The details are left as exercises.

- (a) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- (c) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

HW #2. §6.5 #9b. #10b.

The details are left as exercises.

- #9b. $P^t LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -2 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$
- #10b. $P^t LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -3 & -1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

HW #2. §6.5 #12d.

The details are left as exercises.

	Multiplications/divisions	*Additions/subtractions*
Factoring into LU	$\frac{n^3}{3} - \frac{n}{3}$	$\frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6}$
Solving $Ly^{(k)} = b^{(k)}$	$(\frac{n^2}{2} - \frac{n}{2})m$	$(\frac{n^2}{2} - \frac{n}{2})m$
Solving $Ux^{(k)} = y^{(k)}$	$(\frac{n^2}{2} + \frac{n}{2})m$	$(\frac{n^2}{2} - \frac{n}{2})m$
Total	$\frac{n^3}{3} + mn^2 - \frac{n}{3}$	$\frac{n^3}{3} + (m - \frac{1}{2})n^2 - (m - \frac{1}{6})n$

HW #2. §6.5 #13.

The details are left as exercises.

- (a) To compute $P^t LU$ requires $\frac{n^3}{3} - \frac{n}{3}$ multiplications.
- (b) If $\bar{P}$ is obtained from P by a simple row interchange, then $\det \bar{P} = -\det P$. Thus, if $\bar{P}$ is obtained from P by k interchanges, we have $\det \bar{P} = (-1)^k \det P$.
- (c) Only $n-1$ multiplications are needed in addition to the operations in part (a). That is, totally $\frac{n^3}{3} - \frac{n}{3} + (n-1)$ multiplications are needed.

  If $\det A$ is computed directly by the formula

  $$\det A = \sum_{\sigma \in S_n} sgn(\sigma) \prod_{i=1}^{n} a_{i,\sigma(i)}.$$

  Then totally $(n-1)n!$ multiplications are needed.

- (d) We have $\det A = -741$. Factoring and computing $\det A$ requires 75 Multiplications/Divisions and 55 Additions/Subtractions.