HW8

Textbook §3.2 #6.

Solve

$$
\begin{aligned}
Q_{2,1} &= \frac{(0.5-0.4)Q_{2,0}-(0.5-0.7)Q_{1,0}}{0.7-0.4} \\
Q_{2,2} &= \frac{(0.5-0)Q_{2,1}-(0.5-0.7)Q_{1,1}}{0.7-0} = \frac{27}{7}.
\end{aligned}
$$

Then you will find the solution (Exercise!) $P_2 = f(0.7) = 6.4$.

Textbook §3.2 #8.

$$P_{0,1,2}(1.5) = \frac{(1.5-0)P_{1,2}(1.5) - (1.5-2)P_{0,1}(1.5)}{2-0}$$

$$P_{0,1,2,3}(1.5) = \frac{(1.5-0)P_{1,2,3}(1.5) - (1.5-3)P_{0,1,2}(1.5)}{3-0} = 3.625$$

Textbook §3.2 #12.

Run the following code. Then you will find the solution is approximately 0.567142623527871.

```
format long
x = [0.3      0.4      0.5      0.6     ];
e = [0.740818 0.670320 0.606531 0.548812];
y = e-x;

n = size(x)(2)-1;
Q = zeros(n+1);
Q(:,1) = x';
z = 0;

for i = 1:n
    for j = 1:i
        Q(i+1,j+1) = ( (z-y(i-j+1))*Q(i+1,j) - (z-y(i+1))*Q(i,j) ) ...
                     / (y(i+1)-y(i-j+1));
    end
end
Q(n+1,n+1)
```

```
% without Q
format long
x = [0.3      0.4      0.5      0.6     ];
e = [0.740818 0.670320 0.606531 0.548812];
y = e-x;

n = size(x)(2)-1;
X = zeros(1,(n+2)*(n+1)/2);
Y = zeros(1,(n+2)*(n+1)/2);
X(1:n+1) = x;
Y(1:n+1) = y;
z = 0;

for i = 2:n+1
    for j = 1:i-1
        X(i-j) = ( (z-Y(i-j))*X(i-j+1) - (z-Y(i))*X(i-j) ) ...
                 /(Y(i)-Y(i-j));
    end
end
X(1)
```

Textbook §3.5 #12.

Solve

$$
\begin{aligned}
&S_0(1) = 1\\
&S_0(2) = S_1(2) = 1\\
&S_1(3) = 0\\
&S_0'(2) = S_1'(2)\\
&S_0''(2) = S_1''(2)\\
&S_0''(1) = S_1''(3) = 0.
\end{aligned}
$$

Then you will find the solution (Exercise!) $B = \frac{1}{4}, D = \frac{1}{4}, b = -\frac{1}{2}, d = \frac{1}{4}$.

Textbook §3.5 #13.

Solve

$$\begin{aligned} s_0(2) &= s_1(2) \\ s_0'(2) &= s_1'(2) \\ s_0''(2) &= s_1''(2) \\ s_0'(1) = f'(1) &= f'(3) = s_1'(3). \end{aligned}$$

Then you will find the solution (Exercise!) $a = 4$, $b = 4$, $c = -1$, $d = \frac{1}{3}$.

Textbook §3.5 #14.

Solve

$$
\begin{aligned}
s_0(1) &= s_1(1) \\
s_0'(1) &= s_1'(1) \\
s_0''(1) &= s_1''(1).
\end{aligned}
$$

Then you will find (Exercise!) $B = 0$, $b = -2$. And thus, the solution is $f'(0) = s_0'(0) = B = 0$, $f'(2) = s_1'(2) = b - 8 + 21 = 11$.

Textbook §3.5 #30.

The free cubic spline must be the linear function $L(x)$ through all the data $\{x_i,\ f(x_i)\}_{i=1}^n$ since $L''(x) = 0$ for all x. So properties (a), (b), (c), (d), (e), (f)(i) of Definition 3.10 would be satisfied.

If f is linear, then f is its own clamped cubic spline. If, for example, f satisfies $f(0) = 0, f(1) = 1,$ $f(2) = 2, f'(0) = 1$, and $f'(2) = 0$, then the data lie on a straight line but the function f is not linear. In that case the spline is

$$s(x) = \begin{cases} x - \frac{1}{4}x^2 + \frac{1}{4}x^3, & 0 \le x \le 1 \\ 1 + \frac{5}{4}(x-1) + \frac{1}{2}(x-1)^2 - \frac{3}{4}(x-1)^3, & 1 \le x \le 2 \end{cases}$$

which is not a linear function.

Textbook §3.5 #34.

The five equations are

$$
\begin{aligned}
a_0 &= f(x_0) \\
a_1 &= f(x_1) \\
a_1 &+ b_1(x_2 - x_1) + c_1(x_2 - x_1)^2 = f(x_2) \\
a_0 &+ b_0(x_1 - x_0) + c_0(x_1 - x_0)^2 = a_1 \quad (\because\ S_0(x_1) = S_1(x_1)) \\
b_0 &+ 2c_0(x_1 - x_0) = b_1. \quad (\because\ S_0'(x_1) = S_1'(x_1))
\end{aligned}
$$

If $S \in C^2$, then S is a quadratic on $[x_0,\ x_2]$ (Why?).

Textbook §3.5 #35.

Assume that the quadratic spline s consists of the quadratic polynomials s_0, s_1 mentioned in Problem 34.

Solve

$$\begin{aligned}
s_0(0) &= f(0) \\
s_0(1) &= f(1) \\
s_1(1) &= f(1) \\
s_1(2) &= f(2) \\
s_0'(1) &= s_1'(1) \\
s'(0) &= 2.
\end{aligned}$$

Then you will find the solution (Exercise!)

$$s(x) = \begin{cases} 2x - x^2, & 0 \le x \le 1 \\ 1 + (x-1)^2, & 1 \le x \le 2. \end{cases}$$

HW #3.

There are $3n$ unknowns and $3n - 1$ conditions.

Therefore, the degree is 2 and the number of additional boundary conditions is 1.