Homework Assignment for Week 05

Assigned Oct 13, 2010.

1. Section 2.3: Problems 16, 19, 23(c).

   Hint for problem 16: see problem 4 below.

2. Section 2.4: Problems 8, 9, 10, 12, 14.

3. Read Example 2 and Example 3 of section 2.4 on page 79-81.

4. Suppose that $f(x)$ has a zero of multiplicity m at x^*, compute $g'(x^*)$ where $g(x) = x - f(x)/f'(x)$. What does the result tell you about the order of convergence for Newton's method applied to computing a multiple root? What happens when m becomes larger?

5. Suppose that $f(0) = f'(0) = 0$, $f''(x) > 0$ for all $x \in \mathbb{R}$. Show that $x = 0$ is the unique root of $f(x) = 0$ and Newton's method converges globally to 0, regardless of x_0. What is the order of convergence?

6. Show that the iteration $x_{n+1} = g(x_n)$ converges locally (that is, if x_0 is close enough to a solution) with cubic order ($\alpha = 3$ in Definition 2.6, page 75), provided $g'(x^*) = g''(x^*) = 0$ at a root x^*, $x^* = g(x^*)$. An application to solving $f(x) = 0$ is given in problem 13, section 2.4.