# Homework Assignment for Week 07

Assigned Oct 28, 2009. Due Nov 06, 2009.

1. Section 4.2: Problems 1, 4, 8, 14, 16, 18.

   Hint for problem 8: read problem 9.

   For problem 16, find the largest error in the intervals by sampling with four times as many points. For example, if you interpolated with $n = 10$, then sample on $x_i$, $i = 0, 1, \cdots, 40$ to find the largest interpolation error and report your result. Summarize your result in table(s). Need not hand in the code.

   For problem 18: try it first, then change the left hand side from $f(x) - P_1(x)$ to $f(x) - P_2(x)$ and change the right hand side accordingly. Hand in the $P_2$ version.

2. Section 4.3: Problems 5, 6, 9, 15.

3. Section 4.5: Problems 1, 3, 6, 7, 8, 10.

4. (Programming)

   Section 4.3: Problem 20.

5. Challenge of the week with extra credit. Extension may be possible upon request, pending on your progress.

   Consider solving the system of equations

$$\begin{array}{ll} e^x - 1 + 0.1y & -\alpha = 0 \\ \sin(0.1x + m \sin y) & -\beta = 0 \end{array} \tag{1}$$

   (a) It is clear that $(x, y) = (0, 0)$ is a solution for $\alpha = \beta = 0$. For general $(\alpha, \beta)$ with $\alpha^2 + \beta^2$ small enough, what condition is needed for the existence and uniqueness of solution to equation (1) in the neighborhood of $(0, 0)$? Does equation (1) satisfy this condition when $m = 1$ and $m = -1$?

   (b) What would be the secant method for (1) with $m = 1$?
   Hint: In the 1D case, we solve for $f(x) = 0$ with $f : \mathbb{R} \mapsto \mathbb{R}$. The linear approximation of $f$ is determined by $(x_{n-1}, f(x_{n-1}))$ and $(x_n, f(x_n))$. In case of (1), each equation in (1) is to be approximated by a linear approximation. How many points in $\mathbb{R}^3$ uniquely determines a plane?

   (c) (Programming)
   Solve equation (1) with $m = 1$, $(\alpha, \beta) = (0.1, 0.1)$ using Newton's iteration.

   (d) (Programming)
   Solve equation (1) with $m = 1$, $(\alpha, \beta) = (0.1, 0.1)$ using fixed point iteration.

   (e) (Programming)
   Solve equation (1) with $m = -1$, $(\alpha, \beta) = (0.1, 0.1)$ using fixed point iteration.