Homework Assignment for Week 16

1. Reading instruction for section 7.5 and section 8.1:

   Section 7.5: Study the meaning and derivation of condition number. Skip the 'iterative refinement' part.

   Section 8.1: Study the derivation of linear least square problems, p499-503. Skip the remaining part.

2. Section 7.5: Problems 2(a,c), 9, 10.

3. Use the matlab built-in command 'cond' to find the condition numbers of the Hilbert matrices $H^{(n)}$ in section 7.5, problem 11 for $n = 5, 10, 15, 20$. That is, need not find the inverse of $H^{(n)}$.

4. Section 8.1: Problems 2, 14.

5. Derive the continuous version of least square problem:

   Give n and $f(x) : [0, 1] \mapsto R$, find $a_0, \cdots a_n$ to minimize the quantity

   $$\int_0^1 \left(f(x) - (a_0 + a_1 x + \cdots a_n x^n)\right)^2 \, dx$$

   Derive the normal equation for the coefficient vector $(a_0, \cdots a_n)$.

   Remark: It is a fact that, similar to its discrete counter part in section 8.1, problem 14, both linear least square problem leads to ill-conditioned matrices for large n. Therefore this approach is not recommended for $n > 5$. For your reference, the proper treatment for $n > 5$ can be found in section 8.2.