# Homework Assignment for Week 13

1. Section 4.9: For the integral in Problem 1(a), verify numerically the order of convergence for standard 2nd order (midpoint, trapezoidal) and 4th order (Simpson's, Gaussian with $n = 1$) composite quadratures.

   Take composite midpoint rule for example, find and $p_{\mathrm{M}}$ in $I - I_{h,\mathrm{M}} = O(h^{p_{\mathrm{M}}})$ numerically. Do this for at least one 2nd order method and one 4th order method.

2. Repeat previous problem with any desingularizing method of your choice and verify numerically that the result indeed has 2nd/4th order convergence.

   Remark: simple change of variable $x = t^{-p}$ does not perform well (not enough to restore theoretical order of convergence) for Problem 3 (c,d) due to the oscillatory nature of the integrands. A more subtle subtraction of singular part is required to desingularize the integrand and recover theoretical order of convergence.

3. Section 4.9: Problems 4(a).

   Hint: Without splitting the domain into $\int_0^1 + \int_1^\infty$, the change of variable $t = \frac{1}{x+1}$ will do the trick and is recommended whenever the integrand has no singularity at $x = 0^+$.

4. Section 6.1: Problems 12 (a,b).

   For part (b), change it to $K(x,t) = -\frac{1}{2}e^{-|x-t|}$ and implement Algorithm 6.1 to solve for $u$. For this problem with the new $K$, row interchanging is not needed.

5. Consider an $N^2 \times N^2$ matrix $A$ with $a_{ij} = 0$ except for $i - j = 0, \pm 1, \pm 2$ (only five diagonals have nonzero entries). Estimate total number of multiplication needed for Gaussian elimination without pivoting on $A$ and backward substitution, respectively. Give the leading order of the number of multiplication as $KN^p$. Find $K$ and $p$.

6. Do the same for an $N^2 \times N^2$ matrix $B$ with $b_{ij} = 0$ except for $|i - j| \le N$ (only $2N + 1$ diagonals have nonzero entries).

7. Do the same for an $N^2 \times N^2$ matrix $C$ with $c_{ij} = 0$ except for $i - j = 0, \pm 1$ and $\pm N$ (only 5 separated diagonals have nonzero entries).