Homework Assignment for Week 10

The linear combination $\sum_{i=-k}^{k} c_i f(x_0 + ih)$ is a pth order approximation of $f^{(q)}(x_0)$ if

$$\sum_{i=-k}^{k} c_i f(x_0 + ih) = f^{(q)}(x_0) + O(h^p)$$

1. Section 4.1:

   Apply the round-off error instability calculation (end of section 4.1) to second order approximation of $f''(x_0)$. Find the critical h^* that minimizes $e(h)$. Express both h^* and $e(h^*)$ in terms of the machine ε as $O(\varepsilon^p)$ and find p for both h^* and $e(h^*)$.

2. Section 4.1:

   Repeat the last problem for fourth order approximation of $f'(x_0)$.

3. Section 4.1, 4.2:

   Derive five-point formula for $f'''(x_0)$ and and $f^{(4)}(x_0)$, respectively using $f(x_0)$, $f(x_0 \pm h)$ and $f(x_0 \pm 2h)$. You can either use the method of undetermined coefficients (combined with Taylor expansion around x_0), or apply a variant of Richardson extrapolation on three-point formula for $f'(x_0)$ and $f''(x_0)$, respectively .

4. Section 4.2: Problems 5, 10, 12.

5. For section 4.2, problem 12, also show $K_1 = 0$ alternatively by assuming the expansion

   $$e = (\frac{2+h}{2-h})^{\frac{1}{h}} + C_1 h^{p_1} + \cdots$$

   and find p_1 numerically using $N(h)$ with $h = 0.02$ and 0.01.

6. Section 4.3: Problems 14, 19, 20, 22, 23, 24.