Homework Assignment for Week 07

1. Section 3.1: Problem 6(a). Write a program to evaluate the degree three polynomial using for loops.

   Hint: One can compute

   $$L_{n,k}(x) = \prod_{\substack{i=0,\\ i\neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

   with a for loop in i for each of the $k = 0, \cdots, n$, then evaluate

   $$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}(x)$$

   with another for loop on k. Note that, in C, the index for an array $a(i)$ starts with $i = 0$ by default. However, the index starts with $i = 1$ in matlab. One should shift the index in the Lagrange interpolation formula accordingly.

   Remark: This is not the most efficient method to evaluate the Lagrangian interpolation. The proper way is to evaluate $P(x)$ using Neville's method in section 3.2. An alternative way is to compute the coefficients of $P(x)$ using Newton's divided-difference formula in section 3.3, then evaluate $P(x)$.

2. Section 3.1: Problems 9, 10, 13(a), 17 (the last sentence simply means $h = (10 - 1)/n$ for some positive integer n).

3. Let $x_0, \cdots, x_n$ be uniformly spaced nodes on $[a, b]$ with $x_j = a + jh$, $h = (b - a)/n$.

   (a) Show that $|(x - x_0)\cdots(x - x_n)| \leq n! h^{n+1}$ on $a \leq x \leq b$.

   (b) Let P_n be the degree n interpolating polynomial of e^x with uniformly spaced nodes on $[0, 1]$. Show that

   $$\max_{0\leq x\leq 1} |e^x - P_n(x)| \leq C h^n \to 0 \quad \text{as } n \to \infty$$

   Note that uniform convergence of interpolating polynomials as in (b) does not hold in general. The Illustration on p154-p157 is a good example. In general, we only have

   $$\max_{a\leq x\leq b} |f(x) - P_n(x)| \leq C_n h^n$$

   from Theorem 3.3. A well known example with similar behavior is $f(x) = 1/(1 + x^2)$ on $[-5, 5]$ with uniformly spaced nodes.