

Contents

Preface xi

1 Mathematical Preliminaries and Error Analysis 1

- 1.1 Review of Calculus 2
- 1.2 Round-off Errors and Computer Arithmetic 14
- 1.3 Algorithms and Convergence 29
- 1.4 Numerical Software 38

2 Solutions of Equations in One Variable 47

- 2.1 The Bisection Method 48
- 2.2 Fixed-Point Iteration 55
- 2.3 Newton's Method and Its Extensions 66
- 2.4 Error Analysis for Iterative Methods 78
- 2.5 Accelerating Convergence 86
- 2.6 Zeros of Polynomials and Miller's Method 91
- 2.7 Numerical Software and Chapter Review 101

3 Interpolation and Polynomial Approximation 103

- 3.1 Interpolation and the Lagrange Polynomial 104
- 3.2 Data Approximation and Neville's Method 115
- 3.3 Divided Differences 122
- 3.4 Hermite Interpolation 134
- 3.5 Cubic Spline Interpolation 142
- 3.6 Parametric Curves 162
- 3.7 Numerical Software and Chapter Review 168

4 Numerical Differentiation and Integration 171

- 4.1 Numerical Differentiation 172
- 4.2 Richardson's Extrapolation 183
- 4.3 Elements of Numerical Integration 191

4.4	Composite Numerical Integration	202
4.5	Romberg Integration	211
4.6	Adaptive Quadrature Methods	219
4.7	Gaussian Quadrature	228
4.8	Multiple Integrals	235
4.9	Improper Integrals	250
4.10	Numerical Software and Chapter Review	256

5 Initial-Value Problems for Ordinary Differential Equations 259

5.1	The Elementary Theory of Initial-Value Problems	260
5.2	Euler's Method	266
5.3	Higher-Order Taylor Methods	275
5.4	Runge-Kutta Methods	282
5.5	Error Control and the Runge-Kutta-Fehlberg Method	294
5.6	Multistep Methods	302
5.7	Variable Step-Size Multistep Methods	316
5.8	Extrapolation Methods	323
5.9	Higher-Order Equations and Systems of Differential Equations	331
5.10	Stability	340
5.11	Stiff Differential Equations	349
5.12	Numerical Software	357

6 Direct Methods for Solving Linear Systems 361

6.1	Linear Systems of Equations	362
6.2	Pivoting Strategies	376
6.3	Linear Algebra and Matrix Inversion	386
6.4	The Determinant of a Matrix	400
6.5	Matrix Factorization	406
6.6	Special Types of Matrices	416
6.7	Numerical Software	433

7 Iterative Techniques in Matrix Algebra 437

7.1	Norms of Vectors and Matrices	438
7.2	Eigenvalues and Eigenvectors	450
7.3	The Jacobi and Gauss-Siedel Iterative Techniques	456
7.4	Relaxation Techniques for Solving Linear Systems	469
7.5	Error Bounds and Iterative Refinement	476
7.6	The Conjugate Gradient Method	487
7.7	Numerical Software	503

8 Approximation Theory 505

- 8.1 Discrete Least Squares Approximation 506
- 8.2 Orthogonal Polynomials and Least Squares Approximation 517
- 8.3 Chebyshev Polynomials and Economization of Power Series 526
- 8.4 Rational Function Approximation 535
- 8.5 Trigonometric Polynomial Approximation 545
- 8.6 Fast Fourier Transforms 555
- 8.7 Numerical Software 567

9 Approximating Eigenvalues 569

- 9.1 Linear Algebra and Eigenvalues 570
- 9.2 Orthogonal Matrices and Similarity Transformations 578
- 9.3 The Power Method 585
- 9.4 Householder's Method 602
- 9.5 The QR Algorithm 610
- 9.6 Singular Value Decomposition 624
- 9.7 Numerical Software 638

10 Numerical Solutions of Nonlinear Systems of Equations 641

- 10.1 Fixed Points for Functions of Several Variables 642
- 10.2 Newton's Method 651
- 10.3 Quasi-Newton Methods 659
- 10.4 Steepest Descent Techniques 666
- 10.5 Homotopy and Continuation Methods 674
- 10.6 Numerical Software 682

11 Boundary-Value Problems for Ordinary Differential Equations 685

- 11.1 The Linear Shooting Method 686
- 11.2 The Shooting Method for Nonlinear Problems 693
- 11.3 Finite-Difference Methods for Linear Problems 700
- 11.4 Finite-Difference Methods for Nonlinear Problems 706
- 11.5 The Rayleigh-Ritz Method 712
- 11.6 Numerical Software 728



12 Numerical Solutions to Partial Differential Equations 731

- 12.1 Elliptic Partial Differential Equations 734
- 12.2 Parabolic Partial Differential Equations 743
- 12.3 Hyperbolic Partial Differential Equations 757
- 12.4 An Introduction to the Finite-Element Method 765
- 12.5 Numerical Software 779

Bibliography 781

Answers to Selected Exercises 787

Index 889