1. With $E_1 = (0.25, 0.75)$, $E_2 = (0, 1)$, $E_3 = (0.5, 0.5)$, and $E_4 = (0, 0.5)$, the basis functions are

$$\phi_1(x,y) = \begin{cases} 4x & \text{on } T_1 \\ -2 + 4y & \text{on } T_2, \end{cases}$$

$$\phi_2(x,y) = \begin{cases} -1 - 2x + 2y & \text{on } T_1 \\ 0 & \text{on } T_2, \end{cases}$$

$$\phi_3(x,y) = \begin{cases} 0 & \text{on } T_1 \\ 1 + 2x - 2y & \text{on } T_2, \end{cases}$$

$$\phi_4(x,y) = \begin{cases} 2 - 2x - 2y & \text{on } T_1 \\ 2 - 2x - 2y & \text{on } T_2, \end{cases}$$

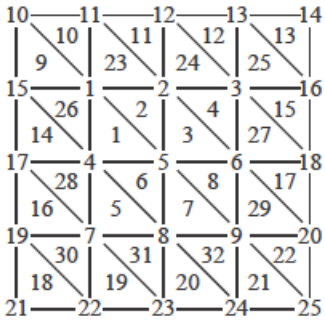
and $\gamma_1 = 0.323825$, $\gamma_2 = 0$, $\gamma_3 = 1.0000$, and $\gamma_4 = 0$.

2. With $E_1 = (0.25, 0.75)$, $E_2 = (0, 1)$, $E_3 = (0.5, 0.5)$, $E_4 = (0, 0.5)$, $E_5 = (0, 0.75)$, and $E_6 = (0.25, 0.5)$, the following results are obtained:

i	j	$a_j^{(i)}$	$b_j^{(i)}$	$c_j^{(i)}$	node
1	1	0	4	0	1
1	2	−3	0	4	2
1	3	4	−4	−4	5
2	1	−2	0	4	1
2	2	−1	4	0	3
2	3	4	−4	−4	6
3	1	0	4	0	1
3	2	3	0	−4	4
3	3	−2	−4	4	5
4	1	−2	0	4	1
4	2	1	−4	0	4
4	3	2	4	−4	6

So $\gamma_1 = 0.3238255$, $\gamma_2 = 0$, $\gamma_3 = 1.0$, $\gamma_4 = 0$, $\gamma_5 = 0$, and $\gamma_6 = 0.5$.

3. The Finite-Element Algorithm with $K = 8, N = 8, M = 32, n = 9, m = 25$, and $NL = 0$ gives the following results, where the labeling is as shown in the diagram.



With the labeling shown in the figure:

$\gamma_1 = 0.511023$, $\gamma_2 = 0.720476$, $\gamma_3 = 0.507899$, $\gamma_4 = 0.720476$,

$\gamma_5 = 1.01885$, $\gamma_6 = 0.720476$, $\gamma_7 = 0.507896$, $\gamma_8 = 0.720476$,

$\gamma_9 = 0.511023$ and $\gamma_i = 0$, for $10 \le i \le 25$

$u(0.125, 0.125) \approx 0.614187$, $u(0.125, 0.25) \approx 0.690343$, $u(0.25, 0.125) \approx 0.690343$, and $u(0.25, 0.25) \approx 0.720476$

4. The Finite-Element Algorithm with $K = 8$, $N = 22$, $M = 32$, $n = 25$, $m = 25$, and $NL = 16$ gives the results shown below, where the labeling is as shown in the figure for Exercise 3:

$\gamma_1 = -0.489695$, $\gamma_2 = 0.0163250$, $\gamma_3 = 0.524243$, $\gamma_4 = 0.0163250$,

$\gamma_5 = 0.00868518$, $\gamma_6 = 0.0163250$, $\gamma_7 = 0.524243$, $\gamma_8 = 0.0163250$,

$\gamma_9 = -0.489695$, $\gamma_{10} = -1.06913$, $\gamma_{11} = -0.684308$, $\gamma_{12} = 0.0581583$,

$\gamma_{13} = 0.752871$, $\gamma_{14} = 0.962801$, $\gamma_{15} = -0.684308$, $\gamma_{16} = 0.752871$,

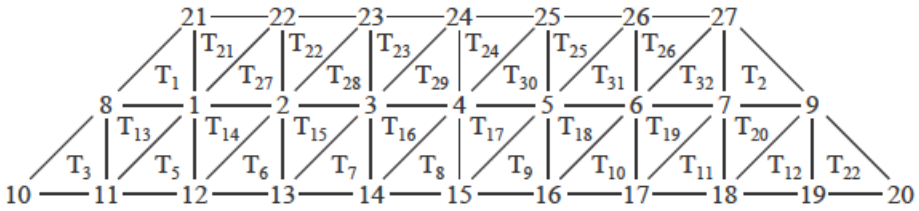
$\gamma_{17} = 0.0581583$, $\gamma_{18} = 0.0581583$, $\gamma_{19} = 0.752871$, $\gamma_{20} = -0.684308$,

$\gamma_{21} = 0.962801$, $\gamma_{22} = 0.752871$, $\gamma_{23} = 0.0581583$, $\gamma_{24} = -0.684308$,

and $\gamma_{25} = -1.06913$.

$u(0.125, 0.125) \approx 0.270284$, $u(0.125, 0.25) \approx -0.238595$, $u(0.25, 0.125) \approx -0.238595$, and $u(0.25, 0.25) \approx 0.0163250$

5. The Finite-Element Algorithm with $K = 0, N = 12, M = 32, n = 20, m = 27$, and $NL = 14$ gives the following results, where the labeling is as shown in the diagram.



$$\begin{aligned}
\gamma_1 &= 21.40335, & \gamma_8 &= 24.19855, & \gamma_{15} &= 20.23334, & \gamma_{22} &= 15,\\
\gamma_2 &= 19.87372, & \gamma_9 &= 24.16799, & \gamma_{16} &= 20.50056, & \gamma_{23} &= 15,\\
\gamma_3 &= 19.10019, & \gamma_{10} &= 27.55237, & \gamma_{17} &= 21.35070, & \gamma_{24} &= 15,\\
\gamma_4 &= 18.85895, & \gamma_{11} &= 25.11508, & \gamma_{18} &= 22.84663, & \gamma_{25} &= 15,\\
\gamma_5 &= 19.08533, & \gamma_{12} &= 22.92824, & \gamma_{19} &= 24.98178, & \gamma_{26} &= 15,\\
\gamma_6 &= 19.84115, & \gamma_{13} &= 21.39741, & \gamma_{20} &= 27.41907, & \gamma_{27} &= 15,\\
\gamma_7 &= 21.34694, & \gamma_{14} &= 20.52179, & \gamma_{21} &= 15
\end{aligned}$$

$u(1,0) \approx 22.92824$, $u(4,0) \approx 22.84663$, and $u\left(\frac{5}{2}, \frac{\sqrt{3}}{2}\right) \approx 18.85895$.