1. The Poisson Equation Finite-Difference Algorithm gives the following results.

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
1	1	0.5	0.5	0.0	0
1	2	0.5	1.0	0.25	0.25
1	3	0.5	1.5	1.0	1

2. The Poisson Equation Finite-Difference Algorithm gives the following results.

i	j	x_i	y_j	w_{ij}	$u(x_i, y_i)$
1	1	1.33333333	0.33333333	0.6348043	0.6359888
1	2	1.33333333	0.66666667	0.7985001	0.7985077
2	1	1.66666667	0.33333333	1.0599924	1.0687200
2	2	1.66666667	0.66666667	1.1698208	1.1700713

3. The Poisson Equation Finite-Difference Algorithm gives the following results.

(a) 30 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
2	2	0.4	0.4	0.1599988	0.16
2	4	0.4	0.8	0.3199988	0.32
4	2	0.8	0.4	0.3199995	0.32
4	4	0.8	0.8	0.6399996	0.64

(b) 29 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
2	1	1.256637	0.3141593	0.2951855	0.2938926
2	3	1.256637	0.9424778	0.1830822	0.1816356
4	1	2.513274	0.3141593	−0.7721948	−0.7694209
4	3	2.513274	0.9424778	−0.4785169	−0.4755283

(c) 126 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
4	3	0.8	0.3	1.2714468	1.2712492
4	7	0.8	0.7	1.7509414	1.7506725
8	3	1.6	0.3	1.6167917	1.6160744
8	7	1.6	0.7	3.0659184	3.0648542

(d) 127 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
2	2	1.2	1.2	0.5251533	0.5250861
4	4	1.4	1.4	1.3190830	1.3189712
6	6	1.6	1.6	2.4065150	2.4064186
8	8	1.8	1.8	3.8088995	3.8088576

4. The Poisson Equation Finite-Difference Algorithm with extrapolation gives the following results.

x_i	y_j	$w_{ij}(h=0.2)$	$w_{ij}(h=0.1)$	$w_{ij}(h=0.05)$	$\text{Ext}1_{i,j}$	$\text{Ext}2_{i,j}$	$\text{Ext}3_{i,j}$
0.4	0.4	0.15999914	0.15999579	0.15998414	0.159994673	0.15998026	0.15997930
0.4	0.8	0.31999888	0.31999384	0.31997558	0.319992160	0.31996949	0.31996798
0.8	0.4	0.31999952	0.31999588	0.31997997	0.319994667	0.31997467	0.31997333
0.8	0.8	0.63999955	0.63999689	0.63998633	0.639996003	0.63998281	0.63998193

5. To incorporate the SOR method, make the following changes to Algorithm 12.1:

STEP 1 Set $h = (b-a)/n$;
$k = (d-c)/m$;
$\omega = 4/\left(2+\sqrt{4-(\cos \pi/m)^2-(\cos \pi/n)^2}\right)$;
$\omega_0 = 1-w$;

In each of Steps 7, 8, 9, 11, 12, 13, 14, 15, and 16 after

set ...

insert

set $E = w_{\alpha,\beta} - z$;
if ($|E| >$ NORM) then set NORM $= |E|$;
set $w_{\alpha,\beta} = \omega_0 E + z$.

where α and β depend on which step is being changed.

6. Using $TOL = 10^{-6}$, the results are the same for both methods. The number of iterations required are listed for each method.

   (a) SOR 14 iterations, Gauss–Seidel 30 iterations, $\omega = 1.259616$

i	j	x_i	y_j	w_{ij}
1	1	0.2	0.2	0.03999975
2	2	0.4	0.4	0.15999994
3	3	0.6	0.6	0.35999994
4	4	0.8	0.8	0.63999998

   (b) SOR 14 iterations, Gauss–Seidel 29 iterations, $\omega = 1.259616$

i	j	x_i	y_j	w_{ij}
2	1	1.256637	0.3141593	0.29518499
2	3	1.256637	0.9424778	0.18308118
4	1	2.513274	0.3141593	−0.77219505
4	3	2.513274	0.9424778	−0.47851735

(c) SOR 30 iterations, Gauss–Seidel 126 iterations, $\omega = 1.527864$

i	j	x_i	y_j	w_{ij}
1	1	0.2	0.1	1.0202140
2	2	0.4	0.2	1.0833400
3	3	0.6	0.3	1.1973456
4	4	0.8	0.4	1.3773776
5	5	1.0	0.5	1.6491565
6	6	1.2	0.6	2.0550775
7	7	1.4	0.7	2.6653128
8	8	1.6	0.8	3.5975766
9	9	1.8	0.9	5.0537432

(d) SOR 30 iterations, Gauss–Seidel 127 iterations, $\omega = 1.527864$

i	j	x_i	y_j	w_{ij}
2	2	1.2	1.2	0.52515626
4	4	1.4	1.4	1.3190907
6	6	1.6	1.6	2.4065227
8	8	1.8	1.8	3.8089025

7. The approximate potential at some typical points gives the following results.

i	j	x_i	y_j	$w_{i,j}$
1	4	0.1	0.4	88
2	1	0.2	0.1	66
4	2	0.4	0.2	66

8. Approximations for the temperature are given in the following table. Convergence was obtained with 293 iterations using the tolerance 10^{-6}.

i	j	x_i	y_j	w_{ij}
5	9	2.0	3.0	5.959624
8	3	3.2	1.0	7.916551
10	9	4.0	3.0	4.679948
12	12	4.8	4.0	2.060342