1. The Nonlinear Finite-Difference method gives the following results.

i	x_i	w_i	$y(x_i)$
1	1.5	0.4067967	0.4054651

2. The Nonlinear Finite-Difference method gives the following results.

i	x_i	w_i	$y(x_i)$
1	−0.75	0.44458639	0.44444444
2	−0.5	0.40015723	0.4
3	−0.25	0.36373152	0.36363636

3. The Nonlinear Finite-Difference Algorithm gives the results in the following tables.

(a)

i	x_i	w_i	$y(x_i)$
2	1.20000000	0.18220299	0.18232156
4	1.40000000	0.33632929	0.33647224
6	1.60000000	0.46988413	0.47000363
8	1.80000000	0.58771808	0.58778666

Convergence in 3 iterations.

(b)

i	x_i	w_i	$y(x_i)$
2	0.31415927	1.36244080	1.36208552
4	0.62831853	1.80138559	1.79999746
6	0.94247780	2.24819259	2.24569937
8	1.25663706	2.59083695	2.58844295

Convergence in 3 iterations.

(c)

i	x_i	w_i	$y(x_i)$
1	0.83775804	0.86205907	0.86205848
2	0.89011792	0.88155964	0.88155882
3	0.94247780	0.89945447	0.89945372
4	0.99483767	0.91579005	0.91578959

Convergence in 2 iterations.

(d)

i	x_i	w_i	$y(x_i)$
4	0.62831853	2.28968807	2.58778525
8	1.25663706	2.41412603	2.95105652
12	1.88495559	2.41412603	2.95105652
16	2.51327412	2.28968807	2.58778525

Convergence in 4 iterations.

4. The Nonlinear Finite-Difference Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
3	1.3	0.4347972	0.4347826
6	1.6	0.3846286	0.3846154
9	1.9	0.3448316	0.3448276

(b)

i	x_i	w_{1i}	$y(x_i)$
3	1.3	2.0694081	2.0692308
6	1.6	2.2250937	2.2250000
9	1.9	2.4263387	2.4263158

(c)

i	x_i	w_{1i}	$y(x_i)$
3	2.3	1.2677078	1.2676917
6	2.6	1.3401418	1.3401268
9	2.9	1.4095432	1.4095383

(d)

i	x_i	w_{1i}	$y(x_i)$
5	1.25	0.4345979	0.4358273
10	1.50	1.3662119	1.3684447
15	1.75	2.9969339	2.9991909

5. (b) For (*4a) the complete results are:

x_i	$w_i(h=0.2)$	$w_i(h=0.1)$	$w_i(h=0.05)$
1.00	0.5000000000	0.5000000000	0.5000000000
1.05			0.4878058215
1.10		0.4761972439	0.4761921720
1.15			0.4651185619
1.20	0.4545886201	0.4545563382	0.4545481813
1.25			0.4444474908
1.30		0.4347956122	0.4347858663
1.35			0.4255352892
1.40	0.4167206681	0.4166802725	0.4166700749
1.45			0.4081666349
1.50		0.4000130439	0.4000032672
1.55			0.3921599714
1.60	0.3846613728	0.3846269650	0.3846182851
1.65			0.3773611389
1.70		0.3703797826	0.3703727277
1.75			0.3636383953
1.80	0.3571694307	0.3571495457	0.3571445322
1.85			0.3508784839
1.90		0.3448311085	0.3448284683
1.95			0.3389835019
2.00	0.3333333333	0.3333333333	0.3333333333

For (4c) we have:

x_i	$w_i(h=0.2)$	$w_i(h=0.1)$	$w_i(h=0.05)$	$EXT_{1,i}$	$EXT_{2,i}$	$EXT_{3,i}$
1.2	2.0340273	2.0335158	2.0333796	2.0333453	2.0333342	2.0333334
1.4	2.1148732	2.1144386	2.1143243	2.1142937	2.1142863	2.1142858
1.6	2.2253630	2.2250937	2.2250236	2.2250039	2.2250003	2.2250000
1.8	2.3557284	2.3556001	2.3555668	2.3555573	2.3555556	2.3355556

6. The approximate deflections using the Nonlinear Finite Difference Algorithm are shown in the following table.

i	x_i	w_i
5	30	0.01028080
10	60	0.01442767
15	90	0.01028080

The results from Exercise 7 in Section 11.3 are:

i	x_i	w_{1i}
5	30	0.0102808
10	60	0.0144277
15	90	0.0102808

Since the results are the same we can conclude that adding the nonlinear term to the differential equation makes no difference.

7. The Jacobian matrix $J = (a_{i,j})$ is tridiagonal with entries given in (11.21). So

$$\begin{aligned}
a_{1,1} &= 2 + h^2 f_y \left(x_1, w_1, \frac{1}{2h}(w_2 - \alpha)\right), \\
a_{1,2} &= -1 + \frac{h}{2} f_{y'} \left(x_1, w_1, \frac{1}{2h}(w_2 - \alpha)\right), \\
a_{i,i-1} &= -1 - \frac{h}{2} f_{y'} \left(x_i, w_i, \frac{1}{2h}(w_{i+1} - w_{i-1})\right), \quad \text{for } 2 \le i \le N-1 \\
a_{i,i} &= 2 + h^2 f_y \left(x_i, w_i, \frac{1}{2h}(w_{i+1} - w_{i-1})\right), \quad \text{for } 2 \le i \le N-1 \\
a_{i,i+1} &= -1 + \frac{h}{2} f_{y'} \left(x_i, w_i, \frac{1}{2h}(w_{i+1} - w_{i-1})\right), \quad \text{for } 2 \le i \le N-1 \\
a_{N,N-1} &= -1 - \frac{h}{2} f_{y'} \left(x_N, w_N, \frac{1}{2h}(\beta - w_{N-1})\right), \\
a_{N,N} &= 2 + h^2 f_y \left(x_N, w_N, \frac{1}{2h}(\beta - w_{N-1})\right).
\end{aligned}$$

Thus, $|a_{i,i}| \ge 2 + h^2\delta$, for $i = 1, \ldots, N$. Since $|f_{y'}(x, y, y')| \le L$ and $h < 2/L$,

$$\left|\frac{h}{2} f_{y'}(x, y, y')\right| \le \frac{hL}{2} < 1.$$

So

$$|a_{1,2}| = \left| -1 + \frac{h}{2} f_{y'} \left(x_1, w_1, \frac{1}{2h}(w_2 - \alpha)\right) \right| < 2 < |a_{1,1}|,$$

$$\begin{aligned}
|a_{i,i-1}| + |a_{i,i+1}| &= -a_{i,i-1} - a_{i,i+1} \\
&= 1 + \frac{h}{2} f_{y'} \left(x_i, w_i, \frac{1}{2h}(w_{i+1} - w_{i-1})\right) + 1 - \frac{h}{2} f_{y'} \left(x_i, w_i, \frac{1}{2h}(w_{i+1} - w_{i-1})\right) \\
&= 2 \le |a_{i,i}|,
\end{aligned}$$

and

$$|a_{N,N-1}| = -a_{N,N-1} = 1 + \frac{h}{2} f_{y'} \left(x_N, w_N, \frac{1}{2h}(\beta - w_{N-1})\right) < 2 < |a_{N,N}|.$$

By Theorem 6.31, the matrix J is nonsingular.