1. The Nonlinear Shooting Algorithm gives $w_1 = 0.405505 \approx \ln 1.5 = 0.405465$.

2. The Nonlinear Shooting Algorithm with $h = 0.25$ requires 4 iterations and gives:

i	x_i	w_{1i}	$y(x_i)$
1	−0.75	0.44444651	0.44444444
2	−0.5	0.40000229	0.4
3	−0.25	0.36363809	0.36363636

3. The Nonlinear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
1	1.1	0.09530982	0.09531018
2	1.2	0.18232094	0.18232156
3	1.3	0.26236347	0.26236426
4	1.4	0.33647129	0.33647224
5	1.5	0.40546403	0.40546511
6	1.6	0.47000243	0.47000363
7	1.7	0.53062693	0.53062825
8	1.8	0.58778522	0.58778666
9	1.9	0.64185232	0.64185389
10	2.0	0.69314549	0.69314718

Convergence in 4 iterations gives $t = 1.0000017$.

(b)

i	x_i	w_{1i}	$y(x_i)$
1	0.15707963	1.16934027	1.16933413
2	0.31415927	1.36209813	1.36208552
3	0.47123890	1.57460167	1.57458304
4	0.62831853	1.80002060	1.79999746
5	0.78539816	2.02814008	2.02811498
6	0.94247780	2.24572329	2.24569937
7	1.09955743	2.43760187	2.43758190
8	1.25663706	2.58845757	2.58844295
9	1.41371669	2.68503045	2.68502044
10	1.57079633	2.71829004	2.71828183

Convergence requires 4 iterations and gives $t = 1.0000301$.

(c)

i	x_i	w_{1i}	$y(x_i)$
1	0.83775804	0.86205941	0.86205848
2	0.89011792	0.88156057	0.88155882
3	0.94247780	0.89945618	0.89945372
4	0.99483767	0.91579268	0.91578959
5	1.04719755	0.93060849	0.93060486

Convergence requires 3 iterations and gives $t = 0.42046725$.

(d)

i	x_i	w_{1i}	$y(x_i)$
1	0.15707963	2.15645346	2.15643446
2	0.31415927	2.30905208	2.30901699
3	0.47123890	2.45403919	2.45399050
4	0.62831853	2.58784539	2.58778525
5	0.78539816	2.70717651	2.70710678
6	0.94247780	2.80909468	2.80901699
7	1.09955743	2.89109072	2.89100652
8	1.25663706	2.95114591	2.95105652
9	1.41371669	2.98778172	2.98768834
10	1.57079633	3.00009624	3.00000000
11	1.72787596	2.98778634	2.98768834
12	1.88495559	2.95115520	2.95105652
13	2.04203522	2.89110479	2.89100652
14	2.19911486	2.80911373	2.80901699
15	2.35619449	2.70720082	2.70710678
16	2.51327412	2.58787536	2.58778525
17	2.67035376	2.45407537	2.45399050
18	2.82743339	2.30909523	2.30901699
19	2.98451302	2.15650454	2.15643446
20	3.14159265	2.00006028	2.00000000

Convergence requires 8 iterations and gives $t = 1.0001253$.

4. The Nonlinear Shooting Algorithm gives the results in the following tables.

   (a) 4 iterations are required, giving:

i	x_i	w_{1i}	$y(x_i)$
3	1.3	0.4347835	0.4347826
6	1.6	0.3846170	0.3846154
9	1.9	0.3448300	0.3448276

   (b) 6 iterations are required, giving:

i	x_i	w_{1i}	$y(x_i)$
3	1.3	2.069249	2.069231
6	1.6	2.225013	2.225000
9	1.9	2.426317	2.426316

   (c) 3 iterations are required, giving:

i	x_i	w_{1i}	$y(x_i)$
3	2.3	1.2676912	1.2676917
6	2.6	1.3401256	1.3401268
9	2.9	1.4095359	1.4095383

   (d) To apply the algorithm we need to redefine the initial value of TK to be 2. Then 7 iterations are required, giving:

i	x_i	w_{1i}	$y(x_i)$
5	1.25	0.4358290	0.4358272
10	1.50	1.3684496	1.3684447
15	1.75	2.9992010	2.9991909

5. (a) Modify Algorithm 11.2 as follows:

STEP 1 Set $h = (b-a)/N$;
$k = 2$;
$TK1 = (\beta - \alpha)/(b-a)$.

STEP 2 Set $w_{1,0} = \alpha$;
$w_{2,0} = TK1$.

STEP 3 For $i = 1, \ldots, N$ do Steps 4 and 5.

STEP 4 Set $x = a + (i-1)h$.

STEP 5 Set

$$k_{1,1} = hw_{2,i-1};$$
$$k_{1,2} = hf(x, w_{1,i-1}, w_{2,i-1});$$
$$k_{2,1} = h(w_{2,i-1} + k_{1,2}/2);$$
$$k_{2,2} = hf(x + h/2, w_{1,i-1} + k_{1,1}/2, w_{2,i-1} + k_{1,2}/2);$$
$$k_{3,1} = h(w_{2,i-1} + k_{2,2}/2);$$
$$k_{3,2} = hf(x + h/2, w_{1,i-1} + k_{2,1}/2, w_{2,i-1} + k_{2,2}/2);$$
$$k_{4,1} = h(w_{2,i-1} + k_{3,2}/2);$$
$$k_{4,2} = hf(x + h/2, w_{1,i-1} + k_{3,1}, w_{2,i-1} + k_{3,2});$$
$$w_{1,i} = w_{1,i-1} + (k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1})/6;$$
$$w_{2,i} = w_{2,i-1} + (k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2})/6.$$

STEP 6 Set $TK2 = TK1 + (\beta - w_{1,N})/(b-a)$.

STEP 7 While $(k \leq M)$ do Steps 8–15.

STEP 8 Set $w_{2,0} = TK2$;
$HOLD = w_{1,N}$.

STEP 9 For $i = 1, \ldots, N$ do Steps 10 and 11.

STEP 10 (Same as *STEP 4*)

STEP 11 (Same as *STEP 5*)

STEP 12 If $|w_{1,N} - \beta| \leq TOL$ then do Steps 13 and 14.

STEP 13 For $i = 0, \ldots, N$ set $x = a + ih$;
OUTPUT$(x, w_{1,i}, w_{2,i})$.

STEP 14 STOP.

STEP 15 Set
$TK = TK2 - (w_{1,N} - \beta)(TK2 - TK1)/(w_{1.N} - HOLD)$;
$TK1 = TK2$;
$TK2 = TK$;
$k = k + 1$.

STEP 16 OUTPUT('Maximum number of iterations exceeded.');
STOP.

(b) For 3(a), 3 iterations give:

i	x_i	w_i	$y(x_i)$
1	1.2	0.45453896	0.45454545
2	1.4	0.41665348	0.41666667
3	1.6	0.38459538	0.38461538
4	1.8	0.35711592	0.35714286

For 3(c), 3 iterations give:

i	x_i	w_i	$y(x_i)$
1	2.2	1.24299575	1.24300281
2	2.4	1.29211897	1.29213540
3	2.6	1.34009800	1.34012683
4	2.8	1.38671706	1.38676227