1. The Linear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
1	0.5	0.82432432	0.82402714

(b)

i	x_i	w_{1i}	$y(x_i)$
1	0.25	0.3937095	0.3936767
2	0.50	0.8240948	0.8240271
3	0.75	1.337160	1.337086

2. The Linear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
1	0.78539816	−0.28245222	−0.28284271

(b)

i	x_i	w_{1i}	$y(x_i)$
1	$\pi/8$	−0.31541496	−0.31543220
2	$\pi/4$	−0.2828507	−0.282842712
3	$3\pi/8$	−0.20718437	−0.20719298

3. The Linear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
3	0.3	0.7833204	0.7831923
6	0.6	0.6023521	0.6022801
9	0.9	0.8568906	0.8568760

(b)

i	x_i	w_{1i}	$y(x_i)$
5	1.25	0.1676179	0.1676243
10	1.50	0.4581901	0.4581935
15	1.75	0.6077718	0.6077740

(c)

i	x_i	w_{1i}	$y(x_i)$
3	0.3	−0.5185754	−0.5185728
6	0.6	−0.2195271	−0.2195247
9	0.9	−0.0406577	−0.0406570

(d)

i	x_i	w_{1i}	$y(x_i)$
3	1.3	0.0655336	0.06553420
6	1.6	0.0774590	0.07745947
9	1.9	0.0305619	0.03056208

4. The Linear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	w_{2i}
1	0.15707963	1.05248506	0.25267869
2	0.31415927	1.07905470	0.08492370
3	0.47123890	1.07905469	−0.08492234
4	0.62831853	1.05248505	−0.25267729

(b)

i	x_i	w_{1i}	w_{2i}
1	0.15707963	−0.06061198	−0.29443007
2	0.31415927	−0.09117479	−0.09251254
3	0.47123890	−0.08959214	0.11091096
4	0.62831853	−0.05748564	0.29239128

(c)

i	x_i	w_{1i}	w_{2i}
5	1.25000000	0.64314227	0.28800448
10	1.50000000	0.68324209	0.07407700
15	1.75000000	0.69226853	0.01166358

(d)

i	x_i	w_{1i}	w_{2i}
3	0.60000000	−0.71219638	−1.82098025
5	1.00000000	−1.64068454	−2.81187530
8	1.60000000	−3.52051591	−2.83551329

5. The Linear Shooting Algorithm with $h = 0.05$ gives the following results.

i	x_i	w_{1i}
6	0.3	0.04990547
10	0.5	0.00673795
16	0.8	0.00033755

The Linear Shooting Algorithm with $h = 0.1$ gives the following results.

i	x_i	w_{1i}
3	0.3	0.05273437
5	0.5	0.00741571
8	0.8	0.00038976

6. For Eq. (11.3), let $u_1(x) = y$ and $u_2(x) = y'$. Then

$$u_1'(x) = u_2(x), \quad a \le x \le b, \quad u_1(a) = \alpha$$

and

$$u_2'(x) = p(x)u_2(x) + q(x)u_1(x) + r(x), \quad a \le x \le b, \quad u_2(a) = 0.$$

For Eq. (11.4), let $v_1(x) = y$ and $v_2(x) = y'$. Then

$$v_1'(x) = v_2(x), \quad a \le x \le b, \quad v_1(a) = 0$$

and

$$v_2'(x) = p(x)v_2(x) + q(x)v_1(x), \quad a \le x \le b, \quad v_2(a) = 1.$$

Using the notation $u_{1,i} = u_1(x_i)$, $u_{2,i} = u_2(x_i)$, $v_{1,i} = v_1(x_i)$ and $v_{2,i} = v_2(x_i)$ leads to the equations in Step 4 of Algorithm 11.1.

7. (a) The approximate potential is $u(3) \approx 36.66702$ using $h = 0.1$.
(b) The actual potential is $u(3) = 36.66667$.

8. Since $y_2(a) = 0$ and $y_2(b) = 0$, the boundary value problem

$$y'' = p(x)y' + q(x)y, \quad a \le x \le b, \quad y(a) = 0, \quad y(b) = 0$$

has $y = 0$ as a unique solution, so $y_2 \equiv 0$.

9. (a) There are no solutions if b is an integer multiple of π and $B \neq 0$.
(b) A unique solution exists whenever b is not an integer multiple of π.
(c) There is an infinite number of solutions if b is an multiple integer of π and $B = 0$.

10. The unique solution is $y(x) = B\left(e^x - e^{-x}\right) / \left(e^b - e^{-b}\right)$. For Exercise 9, we have $q(x) < 0$, so Corollary 11.2 does not apply.