1. The Steepest Descent method gives the following:
   (a) With $\mathbf{x}^{(0)} = (0,0)^t$, we have $\mathbf{x}^{(11)} = (0.4943541, 1.948040)^t$.
   (b) With $\mathbf{x}^{(0)} = (1,1)^t$, we have $\mathbf{x}^{(1)} = (0.50680304, 0.91780051)^t$.
   (c) With $\mathbf{x}^{(0)} = (2,2)^t$, we have $\mathbf{x}^{(1)} = (1.736083, 1.804428)^t$.
   (d) With $\mathbf{x}^{(0)} = (0,0)^t$, we have $\mathbf{x}^{(2)} = (-0.3610092, 0.05788368)^t$.

2. The Steepest Descent method gives the following:
   (a) With $\mathbf{x}^{(0)} = (0,0,0)^t$, we have $\mathbf{x}^{(14)} = (1.043605, 1.064058, 0.9246118)^t$.
   (b) With $\mathbf{x}^{(0)} = (0,0,0)^t$, we have $\mathbf{x}^{(9)} = (0.4932739, 0.9863888, -0.5175964)^t$.
   (c) With $\mathbf{x}^{(0)} = (0,0,0)^t$, we have $\mathbf{x}^{(11)} = (-1.608296, -1.192750, 0.7205642)^t$.
   (d) With $\mathbf{x}^{(0)} = (0,0,0)^t$, we have $\mathbf{x}^{(1)} = (0, 0.00989056, 0.9890556)^t$.

3. The Steepest Descent method with Newton's method gives the following:
   (a) $\mathbf{x}^{(3)} = (0.5, 2)^t$
   (b) $\mathbf{x}^{(3)} = (0.5, 0.8660254)^t$
   (c) $\mathbf{x}^{(4)} = (1.772454, 1.772454)^t$
   (d) $\mathbf{x}^{(3)} = (-0.3736982, 0.05626649)^t$

4. The Steepest Descent method with Newton's method gives the following:
   (a) $\mathbf{x}^{(3)} = (1.036400, 1.085707, 0.9311914)^t$
   (b) $\mathbf{x}^{(3)} = (0.5, 1, -0.5)^t$
   (c) $\mathbf{x}^{(5)} = (-1.456043, -1.664230, 0.4224934)^t$
   (d) $\mathbf{x}^{(6)} = (0.0000000, 0.10000001, 1.0000000)^t$

5. The Steepest Descent method gives the following:
   (a) $\mathbf{x}^{(3)} = (1.036400, 1.085707, 0.9311914)^t$
   (b) $\mathbf{x}^{(3)} = (0.5, 1, -0.5)^t$
   (c) $\mathbf{x}^{(5)} = (-1.456043, -1.664230, 0.4224934)^t$
   (d) $\mathbf{x}^{(6)} = (0.0000000, 0.10000001, 1.0000000)^t$

6. (a) We have $\alpha_1 = 0$, $g_1 = g(x_1, ..., x_n) = g\left(\mathbf{x}^{(0)}\right) = h(\alpha_1)$, $g_3 = g\left(\mathbf{x}^{(0)} - \alpha_3 \nabla g\left(\mathbf{x}^{(0)}\right)\right) = h(\alpha_3)$, $g_2 = g\left(\mathbf{x}^{(0)} - \alpha_2 \nabla g\left(\mathbf{x}^{(0)}\right)\right) = h(\alpha_2)$,

$$\begin{aligned}
h_1 =& \frac{(g_2 - g_1)}{(\alpha_2 - \alpha_1)} = g\left[\mathbf{x}^{(0)} - \alpha_1 \nabla g\left(\mathbf{x}^{(0)}\right), \mathbf{x}^{(0)} - \alpha_2 \nabla g\left(\mathbf{x}^{(0)}\right)\right] = h[\alpha_1, \alpha_2],\\
h_2 =& \frac{(g_3 - g_2)}{(\alpha_3 - \alpha_2)} = g\left[\mathbf{x}^{(0)} - \alpha_2 \nabla g\left(\mathbf{x}^{(0)}\right), \mathbf{x}^{(0)} - \alpha_3 \nabla g\left(\mathbf{x}^{(0)}\right)\right] = h[\alpha_2, \alpha_3],\\
h_3 =& \frac{(h_2 - h_1)}{(\alpha_3 - \alpha_1)}\\
=& g\left[\mathbf{x}^{(0)} - \alpha_1 \nabla g\left(\mathbf{x}^{(0)}\right), \mathbf{x}^{(0)} - \alpha_2 \nabla g\left(\mathbf{x}^{(0)}\right), \mathbf{x}^{(0)} - \alpha_3 \nabla g\left(\mathbf{x}^{(0)}\right)\right] = h[\alpha_1, \alpha_2, \alpha_3].
\end{aligned}$$

The Newton divided-difference form of the second interpolating polynomial is

$$\begin{aligned}
P(\alpha) =& h[\alpha_1] + h[\alpha_1, \alpha_2](\alpha - \alpha_1) + h[\alpha_1, \alpha_2, \alpha_3](\alpha - \alpha_1)(\alpha - \alpha_2)\\
=& g_1 + h_1(\alpha - \alpha_1) + h_3(\alpha - \alpha_1)(\alpha - \alpha_2)\\
=& g_1 + h_1\alpha + h_3\alpha(\alpha - \alpha_2).
\end{aligned}$$

(b) $P'(\alpha) = h_1 - \alpha_2 h_3 + 2h_3\alpha$, so $P'(\alpha) = 0$ when $\alpha = 0.5(\alpha_2 - h_1/h_3)$.