1. Broyden's method gives the following:

   (a) $\mathbf{x}^{(2)} = (0.4777920, 1.927557)^t$
   (b) $\mathbf{x}^{(2)} = (-0.3250070, -0.1386967)^t$
   (c) $\mathbf{x}^{(2)} = (0.5229372, 0.8243491)^t$
   (d) $\mathbf{x}^{(2)} = (1.779500, 1.743396)^t$

2. Broyden's method gives the following:

   (a) $\mathbf{x}^{(2)} = (0.50023123, -1.08029909, -0.52382394)^t$.
   (b) $\mathbf{x}^{(2)} = (-67.005828, 38.314935, 31.690893)^t$.
   (c) $\mathbf{x}^{(2)} = (-1.40360242, -1.67987524, 0.45816509)^t$
   (d) $\mathbf{x}^{(2)} = (0.49840580, -0.19984209, -0.52851353)^t$

3. Broyden's method gives the following:

   (a) With $\mathbf{x}^{(0)} = (0, 0)^t$, we have $\mathbf{x}^{(8)} = (0.5, 2)^t$.
   (b) With $\mathbf{x}^{(0)} = (0, 0)^t$, we have $\mathbf{x}^{(9)} = (-0.3736982, 0.05626649)^t$.
   (c) With $\mathbf{x}^{(0)} = (1, 1)^t$, we have $\mathbf{x}^{(9)} = (0.5, 0.8660254)^t$.
   (d) With $\mathbf{x}^{(0)} = (2, 2)^t$, we have $\mathbf{x}^{(8)} = (1.772454, 1.772454)^t$.

4. Broyden's method gives the following:

   (a) With $\mathbf{x}^{(0)} = (1, 1, 1)^t$, we have $\mathbf{x}^{(18)} = (0.49999953, 0.00319904, -0.52351886)^t$.
   (b) With $\mathbf{x}^{(0)} = (2, 1, -1)^t$, we have $\mathbf{x}^{(10)} = (6.000000000, 1.000000000, -4.000000000)^t$.
   (c) With $\mathbf{x}^{(0)} = (-1, -2, 1)^t$, we have $\mathbf{x}^{(9)} = (-1.456043, -1.664231, 0.4224934)^t$.
   (d) With $\mathbf{x}^{(0)} = (0, 0, 0)^t$, we have $\mathbf{x}^{(5)} = (0.4981447, -0.1996059, -0.5288260)^t$.

5. Broyden's method gives the following:

   (a) With $\mathbf{x}^{(0)} = (2.5, 4)^t$, we have $\mathbf{x}^{(3)} = (2.546947, 3.984998)^t$
   (b) With $\mathbf{x}^{(0)} = (0.11, 0.27)^t$, we have $\mathbf{x}^{(4)} = (0.1212419, 0.2711052)^t$.
   (c) With $\mathbf{x}^{(0)} = (1, 1, 1)^t$, we have $\mathbf{x}^{(3)} = (1.036401, 1.085707, 0.9311914)^t$.
   (d) With $\mathbf{x}^{(0)} = (1, -1, 1)^t$, we have $\mathbf{x}^{(8)} = (0.9, -1, 0.5)^t$; and with $\mathbf{x}^{(0)} = (1, 1, -1)^t$, we have $\mathbf{x}^{(8)} = (0.5, 1, -0.5)^t$.

6. (a) Suppose $(x_1, x_2, x_3, x_4)^t$ is a solution to

$$\begin{aligned} 4x_1 - x_2 + x_3 &= x_1x_4, \\ -x_1 + 3x_2 - 2x_3 &= x_2x_4, \\ x_1 - 2x_2 + 3x_3 &= x_3x_4, \\ x_1^2 + x_2^2 + x_3^2 &= 1. \end{aligned}$$

Multiplying the first three equations by -1 and factoring gives

$$\begin{aligned} 4(-x_1) - (-x_2) + (-x_3) &= (-x_1)x_4, \\ -(-x_1) + 3(-x_2) - 2(-x_3) &= (-x_2)x_4, \\ (-x_1) - 2(-x_2) + 3(-x_3) &= (-x_3)x_4, \\ (-x_1)^2 + (-x_2)^2 + (-x_3)^2 &= 1. \end{aligned}$$

Thus, $(-x_1, -x_2, -x_3, -x_4)^t$ is also a solution.

(b) Using $\mathbf{x}^{(0)} = (1, 1, 1, 1)^t$ gives $\mathbf{x}^{(6)} = (0, 0.70710678, 0.70710678, 1)^t$.
Using $\mathbf{x}^{(0)} = (1, 0, 0, 0)^t$ gives $\mathbf{x}^{(15)} = (0.81649659, 0.40824821, -0.40824837, 3)^t$.
Using $\mathbf{x}^{(0)} = (1, -1, 1, -1)^t$ gives $\mathbf{x}^{(11)} = (0.57735034, -0.57735023, 0.57735025, 6)^t$.
The other three solutions follow easily from part (a).

7. With $\mathbf{x}^{(0)} = (1, 1-1)^t$, Broyden's method gives $\mathbf{x}^{(56)} = (0.5000591, 0.01057235, -0.5224818)^t$.

8. If $\mathbf{z}^t\mathbf{y} = 0$, then $\mathbf{z} = \mathbf{z}_1 + \mathbf{z}_2$, where $\mathbf{z}_1 = \mathbf{0}$ and $\mathbf{z}_2 = \mathbf{z}$. Otherwise, let

$$\mathbf{z}_1 = \frac{\mathbf{y}^t\mathbf{z}}{\|\mathbf{y}\|_2^2}\mathbf{y}$$

be parallel to $\mathbf{y}$ and let $\mathbf{z}_2 = \mathbf{z} - \mathbf{z}_1$. Then

$$\mathbf{z}_2^t\mathbf{y} = \mathbf{z}^t\mathbf{y} - \mathbf{z}_1^t\mathbf{y} = \mathbf{z}^t\mathbf{y} - \left[\frac{\mathbf{y}^t\mathbf{z}}{\mathbf{y}^t\mathbf{y}}\mathbf{y}\right]^t \mathbf{y} = \mathbf{z}^t\mathbf{y} - \frac{\mathbf{z}^t\mathbf{y}}{\mathbf{y}^t\mathbf{y}}\mathbf{y}^t\mathbf{y} = 0.$$

9. Let λ be an eigenvalue of $M = (I + \mathbf{u}\mathbf{v}^t)$ with eigenvector $\mathbf{x} \neq \mathbf{0}$. Then

$$\lambda\mathbf{x} = M\mathbf{x} = \left(I + \mathbf{u}\mathbf{v}^t\right)\mathbf{x} = \mathbf{x} + \left(\mathbf{v}^t\mathbf{x}\right)\mathbf{u}.$$

Thus, $(\lambda - 1)\mathbf{x} = (\mathbf{v}^t\mathbf{x})\,\mathbf{u}$. If $\lambda = 1$, then $\mathbf{v}^t\mathbf{x} = 0$. So $\lambda = 1$ is an eigenvalue of M with multiplicity $n-1$ and eigenvectors $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n-1)}$ where $\mathbf{v}^t\mathbf{x}^{(j)} = 0$, for $j = 1, \ldots, n-1$.
Assuming $\lambda \neq 1$ implies $\mathbf{x}$ and $\mathbf{u}$ are parallel, so for some number α, we have $\mathbf{x} = \alpha\mathbf{u}$. Then

$$(\lambda - 1)\alpha\mathbf{u} = \left(\mathbf{v}^t(\alpha\mathbf{u})\right)\mathbf{u} \quad \text{and} \quad \alpha(\lambda - 1)\mathbf{u} = \alpha\left(\mathbf{v}^t\mathbf{u}\right)\mathbf{u},$$

which implies that

$$\lambda - 1 = \mathbf{v}^t\mathbf{u} \quad \text{or} \quad \lambda = 1 + \mathbf{v}^t\mathbf{u}.$$

Hence, M has eigenvalues λ_i, $1 \leq i \leq n$ where $\lambda_i = 1$, for $i = 1, \ldots, n-1$ and $\lambda_n = 1 + \mathbf{v}^t\mathbf{u}$.
Since $\det M = \prod_{i=1}^{n} \lambda_i$, we have $\det M = 1 + \mathbf{v}^t\mathbf{u}$.

10. (a) Since A^{-1} exists we can write

$$\det\left(A+\mathbf{x}\mathbf{y}^t\right)=\det\left(A+AA^{-1}\mathbf{x}\mathbf{y}^t\right)=\det A\left(I+A^{-1}\mathbf{x}\mathbf{y}^t\right)=\det A\det\left(I+A^{-1}\mathbf{x}\mathbf{y}^t\right).$$

But A^{-1} exists so $\det A\neq 0$. By Exercise 9, $\det\left(I+A^{-1}\mathbf{x}\mathbf{y}^t\right)=1+\mathbf{y}^tA^{-1}\mathbf{x}$. So $\left(A+\mathbf{x}\mathbf{y}^t\right)^{-1}$ exists if and only if $\mathbf{y}^tA^{-1}\mathbf{x}\neq -1$.

(b) Assume $\mathbf{y}^tA^{-1}\mathbf{x}\neq -1$ so that $\left(A+\mathbf{x}\mathbf{y}^t\right)^{-1}$ exists. Therefore,

$$\begin{aligned}\left[A^{-1}-\frac{A^{-1}\mathbf{x}\mathbf{y}^tA^{-1}}{1+\mathbf{y}^tA^{-1}\mathbf{x}}\right]\left(A+\mathbf{x}\mathbf{y}^t\right)=&A^{-1}A-\frac{A^{-1}\mathbf{x}\mathbf{y}^tA^{-1}A}{1+\mathbf{y}^tA^{-1}\mathbf{x}}+A^{-1}\mathbf{x}\mathbf{y}^t-\frac{A^{-1}\mathbf{x}\mathbf{y}^tA^{-1}\mathbf{x}\mathbf{y}^t}{1+\mathbf{y}^tA^{-1}\mathbf{x}}\\ =&I-\frac{A^{-1}\mathbf{x}\mathbf{y}^t}{1+\mathbf{y}^tA^{-1}\mathbf{x}}+A^{-1}\mathbf{x}\mathbf{y}^t-\frac{A^{-1}\mathbf{x}\mathbf{y}^tA^{-1}\mathbf{x}\mathbf{y}^t}{1+\mathbf{y}^tA^{-1}\mathbf{x}}\\ =&I-\frac{A^{-1}\mathbf{x}\mathbf{y}^t-A^{-1}\mathbf{x}\mathbf{y}^t-\mathbf{y}^tA^{-1}\mathbf{x}A^{-1}\mathbf{x}\mathbf{y}^t+A^{-1}\mathbf{x}\mathbf{y}^tA^{-1}\mathbf{x}\mathbf{y}^t}{1+\mathbf{y}^tA^{-1}\mathbf{x}}\\ =&I+\frac{\mathbf{y}^tA^{-1}\mathbf{x}A^{-1}\mathbf{x}\mathbf{y}^t-\mathbf{y}^tA^{-1}\mathbf{x}\left(A^{-1}\mathbf{x}\mathbf{y}^t\right)}{1+\mathbf{y}^tA^{-1}\mathbf{x}}=I.\end{aligned}$$

11. With $\mathbf{x}^{(0)}=(0.75,1.25)^t$, we have $\mathbf{x}^{(4)}=(0.7501948,1.184712)^t$. Thus, $a=0.7501948, b=1.184712$, and the error is 19.796.