1. Newton's method gives the following:

   (a) $\mathbf{x}^{(2)} = (0.4958936, 1.983423)^t$
   (b) $\mathbf{x}^{(2)} = (-0.5131616, -0.01837622)^t$
   (c) $\mathbf{x}^{(2)} = (-23.942626, 7.6086797)^t$
   (d) $\mathbf{x}^{(1)}$ cannot be computed since $J(0)$ is singular.

2. Newton's method gives the following:

   (a) $\mathbf{x}^{(2)} = (0.5001667, 0.2508036, -0.5173874)^t$
   (b) $\mathbf{x}^{(2)} = (4.350877, 18.49123, -19.84211)^t$
   (c) $\mathbf{x}^{(2)} = (1.03668708, 1.08592384, 0.92977932)^t$
   (d) $\mathbf{x}^{(2)} = (0.40716687, 1.30944377, -0.85895477)^t$

3. Graphing in Maple gives the following:

   (a) $(0.5, 0.2)^t$ and $(1.1, 6.1)^t$
   (b) $(-0.35, 0.05)^t, (0.2, -0.45)^t, (0.4, -0.5)^t$ and $(1, -0.3)^t$
   (c) $(-1, 3.5)^t$, $(2.5, 4)^t$
   (d) $(0.11, 0.27)^t$

4. Graphing in Maple gives the following:

   (a) $(0.5, 0.5, -0.5)^t$
   (b) $(7, -1, -2)^t$
   (c) $(1, 1, 1)^t$
   (d) $(1, -1, 1)^t$ and $(1, 1, -1)^t$

5. Newton's method gives the following:

   (a) With $\mathbf{x}^{(0)} = (0.5, 2)^t, \mathbf{x}^{(3)} = (0.5, 2)^t$ With $\mathbf{x}^{(0)} = (1.1, 6.1), \mathbf{x}^{(3)} = (1.0967197, 6.0409329)^t$
   (b) With $\mathbf{x}^{(0)} = (-0.35, 0.05)^t, \mathbf{x}^{(3)} = (-0.37369822, 0.056266490^t$ With $\mathbf{x}^{(0)} = (0.2, -0.45)^t, \mathbf{x}^{(4)} = (0.14783924, -0.43617762)^t$ With $\mathbf{x}^{(0)} = (0.4, -0.5)^t, \mathbf{x}^{(3)} = (0.40809566, -0.49262939)^t$ With $\mathbf{x}^{(0)} = (1, -0.3)^t, \mathbf{x}^{(4)} = (1.0330715, -0.27996184)^t$
   (c) With $\mathbf{x}^{(0)} = (-1, 3.5)^t$, $\mathbf{x}^{(1)} = (-1, 3.5)^t$ and $\mathbf{x}^{(0)} = (2.5, 4)^t$, $\mathbf{x}^{(3)} = (2.546947, 3.984998)^t$.
   (d) With $\mathbf{x}^{(0)} = (0.11, 0.27)^t$, $\mathbf{x}^{(6)} = (0.1212419, 0.2711051)^t$.

6. Newton's method gives the following:

   (a) $\mathbf{x}^{(12)} = (0.49999953, 0.00319906, -0.52351886)^t$
   (b) $\mathbf{x}^{(4)} = (6.17107462, -1.08216201, -2.08891251)^t$
   (c) With $\mathbf{x}^{(0)} = (1,1,1)^t$, $\mathbf{x}^{(3)} = (1.036401, 1.085707, 0.9311914)^t$.
   (d) With $\mathbf{x}^{(0)} = (1,-1,1)^t$, $\mathbf{x}^{(5)} = (0.9,-1,0.5)^t$; and with $\mathbf{x}^{(0)} = (1,-1,1)^t$, $\mathbf{x}^{(5)} = (0.5,1,-0.5)^t$.

7. Newton's method gives the following:

   (a) $\mathbf{x}^{(5)} = (0.5000000, 0.8660254)^t$
   (b) $\mathbf{x}^{(6)} = (1.772454, 1.772454)^t$
   (c) $\mathbf{x}^{(5)} = (-1.456043, -1.664230, 0.4224934)^t$
   (d) $\mathbf{x}^{(4)} = (0.4981447, -0.1996059, -0.5288260)^t$

8. (a) Suppose $(x_1, x_2, x_3, x_4)^t$ is a solution to

$$\begin{aligned} 4x_1 - x_2 + x_3 &= x_1x_4, \\ -x_1 + 3x_2 - 2x_3 &= x_2x_4, \\ x_1 - 2x_2 + 3x_3 &= x_3x_4, \\ x_1^2 + x_2^2 + x_3^2 &= 1. \end{aligned}$$

   Multiplying the first three equations by -1 and factoring gives

$$\begin{aligned} 4(-x_1) - (-x_2) + (-x_3) &= (-x_1)x_4, \\ -(-x_1) + 3(-x_2) - 2(-x_3) &= (-x_2)x_4, \\ (-x_1) - 2(-x_2) + 3(-x_3) &= (-x_3)x_4, \\ (-x_1)^2 + (-x_2)^2 + (-x_3)^2 &= 1. \end{aligned}$$

   Thus, $(-x_1, -x_2, -x_3, x_4)^t$ is also a solution.

   (b) Using $\mathbf{x}^{(0)} = (1,1,1,1)^t$ gives $\mathbf{x}^{(5)} = (0, 0.70710678, 0.70710678, 1)^t$.
   Using $\mathbf{x}^{(0)} = (1,0,0,0)^t$ gives $\mathbf{x}^{(6)} = (0.81649658, 0.40824829, -0.40824829, 3)^t$.
   Using $\mathbf{x}^{(0)} = (1,-1,1,-1)^t$ gives $\mathbf{x}^{(5)} = (0.57735027, -0.57735027, 0.57735027, 6)^t$.
   The other three solutions follow easily from part (a).

9. With $\mathbf{x}^{(0)} = (1, 1-1)^t$ and $TOL = 10^{-6}$, we have $\mathbf{x}^{(20)} = (0.5, 9.5\times10^{-7}, -0.5235988)^t$.

10. Since $f_j(x_1, \ldots, x_n) = a_{j1}x_1 + a_{j2}x_2 + \ldots + a_{jn}x_n - b_j$, we have $\dfrac{\partial f_j}{\partial x_1} = a_{ji}$. Hence,

$$J(\mathbf{x}) = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} = A.$$

Further,

$$\mathbf{F}\left(\mathbf{x}^{(0)}\right) = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = J\left(\mathbf{x}^{(0)}\right)\mathbf{x}^{(0)} - \mathbf{b}.$$

Thus, given $\mathbf{x}^{(0)}$, we have

$$\begin{aligned} \mathbf{x}^{(1)} &= \mathbf{x}^{(0)} - J\left(\mathbf{x}^{(0)}\right)^{-1}\left(J\left(\mathbf{x}^{(0)}\right)\mathbf{x}^{(0)} - \mathbf{b}\right) \\ &= \mathbf{x}^{(0)} - J\left(\mathbf{x}^{(0)}\right)^{-1} J\left(\mathbf{x}^{(0)}\right)\mathbf{x}^{(0)} + J\left(\mathbf{x}^{(0)}\right)^{-1}\mathbf{b} = J\left(\mathbf{x}^{(0)}\right)^{-1}\mathbf{b} = A^{-1}\mathbf{b}. \end{aligned}$$

So given any $\mathbf{x}^{(0)}$, the solution to the linear system is $\mathbf{x}^{(1)}$.

11. When the dimension n is 1, $\mathbf{F}(\mathbf{x})$ is a one-component function $f(\mathbf{x}) = f_1(\mathbf{x})$, and the vector $\mathbf{x}$ has only one component $x_1 = x$. In this case, the Jacobian matrix $J(\mathbf{x})$ reduces to the 1×1 matrix $[\partial f_1/\partial x_1(\mathbf{x})] = f'(\mathbf{x}) = f'(x)$. Thus, the vector equation

$$\mathbf{x}^{(k)} = \mathbf{x}^{(k-1)} - J(\mathbf{x}^{(k-1)})^{-1}\mathbf{F}(\mathbf{x}^{(k-1)})$$

becomes the scalar equation

$$x_k = x_{k-1} - f(x_{k-1})^{-1} f(x_{k-1}) = x_{k-1} - \frac{f(x_{k-1})}{f'(x_{k-1})}.$$

12. The constants required for the pressure equation are in part (a). The approximate radius is in part (b).

(a) $k_1 = 8.77125, k_2 = 0.259690, k_3 = -1.37217$

(b) Solving the equation

$$\frac{500}{\pi r^2} = k_1 e^{k_2 r} + k_3 r$$

numerically, gives $r = 3.18517$.

13. With $\theta_i^{(0)} = 1$, for each $i = 1, 2, \ldots, 20$, the following results are obtained.

i	1	2	3	4	5	6
$\theta_i^{(5)}$	0.14062	0.19954	0.24522	0.28413	0.31878	0.35045

i	7	8	9	10	11	12	13
$\theta_i^{(5)}$	0.37990	0.40763	0.43398	0.45920	0.48348	0.50697	0.52980

i	14	15	16	17	18	19	20
$\theta_i^{(5)}$	0.55205	0.57382	0.59516	0.61615	0.63683	0.65726	0.67746

14. (a) We have

$$\frac{\partial E}{\partial a} = 2\sum_{i=1}^{n}\left(w_i y_i - \frac{a}{(x_i - b)^c}\right)\left(\frac{1}{(x_i - b)^c}\right) = 0,$$

$$\frac{\partial E}{\partial b} = 2\sum_{i=1}^{n}\left(w_i y_i - \frac{a}{(x_i - b)^c}\right)\left(\frac{-ac}{(x_i - b)^{c+1}}\right) = 0,$$

and

$$\frac{\partial E}{\partial c} = 2\sum_{i=1}^{n}\left(w_i y_i - \frac{a}{(x_i - b)^c}\right)\ln(x_i - b)\left(\frac{-a}{(x_i - b)^c}\right) = 0.$$

Solving for a in the first equation and substituting into the second and third equations gives the linear system.

(b) With $\mathbf{x}^{(0)} = (26.8, 8.3)^t = (b_0, c_0)^t$, we have $\mathbf{x}^{(7)} = (26.77021, 8.451831)^t$. Thus, $a = 2.217952 \times 10^6$, $b = 26.77021$, $c = 8.451831$, and

$$\sum_{i=1}^{n}\left(w_i y_i - \frac{a}{(x_i - b)^c}\right)^2 = 0.7821139.$$