1. The approximate eigenvalues and approximate eigenvectors are:

   (a) $\mu^{(3)} = 3.666667, \quad \mathbf{x}^{(3)} = (0.9772727, 0.9318182, 1)^t$
   (b) $\mu^{(3)} = 2.000000, \quad \mathbf{x}^{(3)} = (1, 1, 0.5)^t$
   (c) $\mu^{(3)} = 5.000000, \quad \mathbf{x}^{(3)} = (-0.2578947, 1, -0.2842105)^t$
   (d) $\mu^{(3)} = 5.038462, \quad \mathbf{x}^{(3)} = (1, 0.2213741, 0.3893130, 0.4045802)^t$

2. The approximate eigenvalues and approximate eigenvectors are:

   (a) $\mu^{(3)} = 6.0508475, \quad \mathbf{x}^{(3)} = (1, 0.57142857, 0.77591036)^t$
   (b) $\mu^{(3)} = 5.5263158, \quad \mathbf{x}^{(3)} = (0.17117117, 0.45945946, 1, 0.9459460)^t$
   (c) $\mu^{(3)} = 7.531073, \quad \mathbf{x}^{(3)} = (0.6886722, -0.6706677, -0.9219805, 1)^t$
   (d) $\mu^{(3)} = 4.106061, \quad \mathbf{x}^{(3)} = (0.1254613, 0.08487085, 0.00922509, 1)^t$

3. The approximate eigenvalues and approximate eigenvectors are:

   (a) $\mu^{(3)} = 1.027730, \quad \mathbf{x}^{(3)} = (-0.1889082, 1, -0.7833622)^t$
   (b) $\mu^{(3)} = -0.4166667, \quad \mathbf{x}^{(3)} = (1, -0.75, -0.6666667)^t$
   (c) $\mu^{(3)} = 17.64493, \quad \mathbf{x}^{(3)} = (-0.3805794, -0.09079132, 1)^t$
   (d) $\mu^{(3)} = 1.378684, \quad \mathbf{x}^{(3)} = (-0.3690277, -0.2522880, 0.2077438, 1)^t$

4. The approximate eigenvalues and approximate eigenvectors are:

   (a) $\mu^{(3)} = 5.9182329, \quad \mathbf{x}^{(3)} = (1, 0.55263364, 0.81296561)^t$
   (b) $\mu^{(3)} = 2.6458436, \quad \mathbf{x}^{(3)} = (0.60846040, 1, -0.326774888, 0.03738318)^t$
   (c) $\mu^{(3)} = 3.996073, \quad \mathbf{x}^{(3)} = (0.9991429, 0.9932014, 1, 0.9939825)^t$
   (d) $\mu^{(3)} = 4.105293, \quad \mathbf{x}^{(3)} = (0.06281419, 0.08704089, 0.01825213, 1)^t$

5. The approximate eigenvalues and approximate eigenvectors are:

   (a) $\mu^{(3)} = 3.959538, \quad \mathbf{x}^{(3)} = (0.5816124, 0.5545606, 0.5951383)^t$
   (b) $\mu^{(3)} = 2.0000000, \quad \mathbf{x}^{(3)} = (-0.6666667, -0.6666667, -0.3333333)^t$
   (c) $\mu^{(3)} = 7.189567, \quad \mathbf{x}^{(3)} = (0.5995308, 0.7367472, 0.3126762)^t$
   (d) $\mu^{(3)} = 6.037037, \quad \mathbf{x}^{(3)} = (0.5073714, 0.4878571, -0.6634857, -0.2536857)^t$

6. The approximate eigenvalues and approximate eigenvectors are:

   (a) $\mu^{(3)} = 3.8484163, \quad \mathbf{x}^{(3)} = (0.29841319, -0.46893501, 0.8312939)^t$
   (b) $\mu^{(3)} = 4.6905660, \quad \mathbf{x}^{(3)} = (-0.95557266, -0.29122214, 0.04550346)^t$
   (c) $\mu^{(3)} = 5.142562, \quad \mathbf{x}^{(3)} = (0.8373051, 0.3701770, 0.1939022, 0.3525495)^t$
   (d) $\mu^{(3)} = 8.593142, \quad \mathbf{x}^{(3)} = (-0.4134762, 0.4026664, 0.5535536, -0.6003962)^t$

7. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(9)} = 3.999908, \quad x^{(9)} = (0.9999943, 0.9999828, 1)^t$
(b) $\mu^{(13)} = 2.414214, \quad x^{(13)} = (1, 0.7071429, 0.7070707)^t$
(c) $\mu^{(9)} = 5.124749, \quad x^{(9)} = (-0.2424476, 1, -0.3199733)^t$
(d) $\mu^{(24)} = 5.235861, \quad x^{(24)} = (1, 0.6178361, 0.1181667, 0.4999220)^t$

8. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(12)} = 5.9193476, \quad x^{(12)} = (1, 0.55478845, 0.80995816)^t$
(b) $\mu^{(14)} = 5.6658972, \quad x^{(14)} = (0.05520444, 0.25749728, 1, 0.88861726)^t$
(c) $\mu^{(17)} = 8.999667, \quad x^{(17)} = (0.9999085, -0.9999078, -0.9999993, 1)^t$
(d) The method did not converge in 25 iterations. However, $\lambda_1 \approx \mu^{(363)} = 4.105309$, then$x^{(363)} = (0.06286299, 0.08702754, 0.01824680, 1)^t$ and $\lambda_2 \approx \mu^{(15)} = -4.024308$

9. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(9)} = 1.00001523$ with $x^{(9)} = (-0.19999391, 1, -0.79999087)^t$
(b) $\mu^{(12)} = -0.41421356$ with $x^{(12)} = (1, -0.70709184, -0.707121720^t$
(c) The method did not converge in 25 iterations. However, convergence occurred with $\mu^{(42)} = 1.63663642$ with $x^{(42)} = (-0.57068151, 0.3633658, 1)^t$
(d) $\mu^{(9)} = 1.38195929$ with $x^{(9)} = (-0.38194003, -0.23610068, 0.23601909, 1)^t$

10. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(7)} = 5.9196688, \quad x^{(7)} = (1, 0.55484776, 0.80997330)^t$
(b) $\mu^{(6)} = 2.6459312, \quad x^{(6)} = (0.60756191, 1, -0.32506930, 0.03836926)^t$
(c) $\mu^{(6)} = 3.999997, \quad x^{(6)} = (0.9999939, 0.9999999, 0.9999940, 1)^t$
(d) $\mu^{(3)} = 4.105293, \quad x^{(3)} = (0.06281419, 0.08704089, 0.01825213, 1)^t$

11. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(8)} = 4.0000000, \quad x^{(8)} = (0.5773547, 0.5773282, 0.5773679)^t$
(b) $\mu^{(13)} = 2.414214, \quad x^{(13)} = (-0.7071068, -0.5000255, -0.4999745)^t$
(c) $\mu^{(16)} = 7.223663, \quad x^{(16)} = (0.6247845, 0.7204271, 0.3010466)^t$
(d) $\mu^{(20)} = 7.086130, \quad x^{(20)} = (0.3325999, 0.2671862, -0.7590108, -0.4918246)^t$

12. The approximate eigenvalues and approximate eigenvectors are:

(a) The method did not converge in 25 iterations. Dominant eigenvalues are $\sqrt{15}$ and $-\sqrt{15}$.
(b) $\mu^{(16)} = 4.8347780, \quad x^{(16)} = (-0.92904870, -0.36778361, 0.04004662)^t$
(c) $\mu^{(21)} = 5.236068, \quad x^{(21)} = (0.7795539, 0.4815996, 0.09214214, 0.3897016)^t$
(d) $\mu^{(16)} = 9.0000000, \quad x^{(16)} = (-0.4999592, 0.4999584, 0.5000408, -0.5000416)^t$

13. The approximate eigenvalues and approximate eigenvectors are:

(a) $\lambda_2 \approx \mu^{(1)} = 1.000000$, $\quad \mathbf{x}^{(1)} = (-2.999908, 2.999908, 0)^t$

(b) $\lambda_2 \approx \mu^{(1)} = 1.000000$, $\quad \mathbf{x}^{(1)} = (0, -1.414214, 1.414214)^t$

(c) $\lambda_2 \approx \mu^{(6)} = 1.636734$, $\quad \mathbf{x}^{(6)} = (1.783218, -1.135350, -3.124733)^t$

(d) $\lambda_2 \approx \mu^{(10)} = 3.618177$, $\quad \mathbf{x}^{(10)} = (0.7236390, -1.170573, 1.170675, -0.2763374)^t$

14. The approximate eigenvalues and approximate eigenvectors are:

(a) The method did not converge in 25 iterations. The remaining eigenvalues are complex numbers.

(b) $\mu^{(9)} = 2.6459095$, $\quad \mathbf{x}^{(9)} = (-1.6930953, -2.7867383, 0.90582533, -0.10692842)^t$

(c) $\lambda_2 \approx \mu^{(21)} = 5.000051$, $\quad \mathbf{x}^{(21)} = (1.999338, -1.999603, 1.999603, -2.000198)^t$

(d) $\mathbf{x}^{(15)} = (-8.151965, 2.100699, 0.7519080, -0.3554941)^t$.

15. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(8)} = 4.000001$, $\quad \mathbf{x}^{(8)} = (0.9999773, 0.99993134, 1)^t$

(b) The method fails because of division by zero.

(c) $\mu^{(7)} = 5.124890$, $\quad \mathbf{x}^{(7)} = (-0.2425938, 1, -0.3196351)^t$

(d) $\mu^{(15)} = 5.236112$, $\quad \mathbf{x}^{(15)} = (1, 0.6125369, 0.1217216, 0.4978318)^t$

16. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(9)} = 5.91971410$, $\quad \mathbf{x}^{(9)} = (1, 0.55478845, 0.80995816)^t$

(b) $\mu^{(11)} = 5.66581211$, $\quad \mathbf{x}^{(11)} = (0.0552044, 0.25749928, 1, 0.88861728)^t$

(c) $\mu^{(10)} = 8.999890$, $\quad \mathbf{x}^{(10)} = (0.9944137, -0.9942148, -0.9997991, 1)^t$

(d) $\mu^{(11)} = 4.105317$, $\quad \mathbf{x}^{(11)} = (0.11716540, 0.072853995, 0.01316655, 1)^t$

17. The approximate eigenvalues and approximate eigenvectors are:

(a) $\mu^{(2)} = 1.000000$, $\quad \mathbf{x}^{(2)} = (0.1542373, -0.7715828, 0.6171474)^t$

(b) $\mu^{(13)} = 1.000000$, $\quad \mathbf{x}^{(13)} = (0.00007432, -0.7070723, 0.7071413)^t$

(c) $\mu^{(14)} = 4.961699$, $\quad \mathbf{x}^{(14)} = (-0.4814472, 0.05180473, 0.8749428)^t$

(d) $\mu^{(17)} = 4.428007$, $\quad \mathbf{x}^{(17)} = (0.7194230, 0.4231908, 0.1153589, 0.5385466)^t$

18. The Power method was applied to the matrices in Exercise 1 using $\mathbf{x}^{(0)}$ as given with $TOL = 10^{-4}$. The following table summarizes the results. (*Note:* The results are very sensitive to roundoff error.)

	λ_1	Number of iterations	λ_2	Eigenvector
(1a)	3.999908	2	1.000037	$(-0.1999411, 1, -0.799911)^t$
(1b)	2.414213562	15	1.000003	$(0.00004881, -0.9999485, 1)^t$
(1c)	5.12488541	5	1.636636	$(-0.5706569, 0.3633325, 1)^t$
(1d)	5.23606796	13	3.617997	$(-0.6180177, 1, -0.9999990, 0.2360394)^t$

19. (a) We have $|\lambda| \leq 6$ for all eigenvalues λ.

(b) The approximate eigenvalue is $\mu^{(133)} = 0.69766854$, with the approximate eigenvector $\mathbf{x}^{(133)} = (1, 0.7166727, 0.2568099, 0.04601217)^t$.

(c) The characteristic polynomial is

$$P(\lambda) = \lambda^4 - \frac{1}{4}\lambda - \frac{1}{16},$$

and the eigenvalues are

$$\lambda_1 = 0.6976684972, \quad \lambda_2 = -0.2301775942+0.56965884i, \quad \lambda_3 = -0.2301775942-0.56965884i,$$

and

$$\lambda_4 = -0.237313308.$$

(d) The beetle population should approach zero since A is convergent.

20. Since

$$\mathbf{x}^t = \frac{1}{\lambda_1 v_i^{(1)}}(a_{i1}, a_{i2}, \ldots, a_{in}),$$

the ith row of B is

$$(a_{i1}, a_{i2}, \ldots, a_{in}) - \frac{\lambda_1}{\lambda_1 v_i^{(1)}}\left(v_i^{(1)}a_{i1}, v_i^{(1)}a_{i2}, \ldots, v_i^{(1)}a_{in}\right) = \mathbf{0}.$$

21. Using the Inverse Power method with $\mathbf{x}^{(0)} = (1, 0, 0, 1, 0, 0, 1, 0, 0, 1)^t$ and $q = 0$ gives the following results:

(a) $\mu^{(49)} = 1.0201926$, so $\rho(A^{-1}) \approx 1/\mu^{(49)} = 0.9802071$;

(b) $\mu^{(30)} = 1.0404568$, so $\rho(A^{-1}) \approx 1/\mu^{(30)} = 0.9611163$;

(c) $\mu^{(22)} = 1.0606974$, so $\rho(A^{-1}) \approx 1/\mu^{(22)} = 0.9427760$.
The method appears to be stable for all α in $[\frac{1}{4}, \frac{3}{4}]$.

22. (a) $\rho\left(A^{-1}\right) = 0.9801485$
    (b) $\rho\left(A^{-1}\right) = 0.9610699$
    (c) $\rho\left(A^{-1}\right) = 0.9427198$
    The method appears to be stable for $\alpha > 0$.

23. Forming $A^{-1}B$ and using the Power method with $\mathbf{x}^{(0)} = (1, 0, 0, 1, 0, 0, 1, 0, 0, 1)^t$ gives the following results:

    (a) The spectral radius is approximately $\mu^{(46)} = 0.9800021$.
    (b) The spectral radius is approximately $\mu^{(25)} = 0.9603543$.
    (c) The spectral radius is approximately $\mu^{(18)} = 0.9410754$.

24. (a) $\lambda_1 = -6$, $\lambda_2 = -5$, $\lambda_3 = -2$, the system is stable.
    (b) $\lambda_1 = -2$, $\lambda_2 = -1.1067711$, $\lambda_3 = -3.94664 + 0.82970i$, $\lambda_4 = -3.94664 - 0.82970i$, the system is stable.