1. The trigonometric interpolating polynomials are:

   (a) $S_2(x) = -12.33701 + 4.934802\cos x - 2.467401\cos 2x + 4.934802\sin x$

   (b) $S_2(x) = -6.168503 + 9.869604\cos x - 3.701102\cos 2x + 4.934802\sin x$

   (c) $S_2(x) = 1.570796 - 1.570796\cos x$

   (d) $S_2(x) = -0.5 - 0.5\cos 2x + \sin x$

2. Parts (a) and (b) give the same answer: The trigonometric interpolating polynomial is

$$\begin{aligned} S_4(x) = & -4.626377 + 6.679518\cos x - 3.701102\cos 2x + 3.190086\cos 3x - 1.542126\cos 4x \\ & + 5.956833\sin x - 2.467401\sin 2x + 1.022031\sin 3x. \end{aligned}$$

3. The Fast Fourier Transform Algorithm gives the following trigonometric interpolating polynomials.

   (a) $S_4(x) = -11.10331 + 2.467401\cos x - 2.467401\cos 2x + 2.467401\cos 3x - 1.233701\cos 4x + 5.956833\sin x - 2.467401\sin 2x + 1.022030\sin 3x$

   (b) $S_4(x) = 1.570796 - 1.340759\cos x - 0.2300378\cos 3x$

   (c) $S_4(x) = -0.1264264 + 0.2602724\cos x - 0.3011140\cos 2x + 1.121372\cos 3x + 0.04589648\cos 4x - 0.1022190\sin x + 0.2754062\sin 2x - 2.052955\sin 3x$

   (d) $S_4(x) = -0.1526819 + 0.04754278\cos x + 0.6862114\cos 2x - 1.216913\cos 3x + 1.176143\cos 4x - 0.8179387\sin x + 0.1802450\sin 2x + 0.2753402\sin 3x$

4. (a) The trigonometric interpolating polynomial is

$$\begin{aligned} S_4(x) =& 0.1735500 - 0.02475498\cos(2x-1)\pi - 0.0697570\cos 2(2x-1)\pi \\ & + 0.08468317\cos 3(2x-1)\pi - 0.04386479\cos 4(2x-1)\pi + 0.2268260\sin(2x-1)\pi \\ & - 0.1021640\sin 2(2x-1)\pi + 0.04284648\sin 3(2x-1)\pi. \end{aligned}$$

   (b) 0.1735500

   (c) 0.2232443

5. The trigonometric polynomials give the following integral approximations.

	Approximation	Actual
(a)	−69.76415	−62.01255
(b)	9.869602	9.869604
(c)	−0.7943605	−0.2739383
(d)	−0.9593287	−0.9557781

6. The b_k terms are all zero. The a_k terms are

$a_0 = -4.01287586$, $a_1 = 3.80276903$, $a_2 = -2.23519870$, $a_3 = 0.63810403$, $a_4 = -0.31550821$, $a_5 = 0.19408145$, $a_6 = -0.13464491$, $a_7 = 0.10100593$, $a_8 = -0.08015708$, $a_9 = 0.06643598$, $a_{10} = -0.05704353$, $a_{11} = 0.05046675$, $a_{12} = -0.04583431$, $a_{13} = 0.04262318$, $a_{14} = -0.04051395$, $a_{15} = 0.03931584$, and $a_{16} = -0.03892713$.

7. The b_j terms are all zero. The a_j terms are as follows:

$a_0 = -4.0008033$	$a_1 = 3.7906715$	$a_2 = -2.2230259$	$a_3 = 0.6258042$
$a_4 = -0.3030271$	$a_5 = 0.1813613$	$a_6 = -0.1216231$	$a_7 = 0.0876136$
$a_8 = -0.0663172$	$a_9 = 0.0520612$	$a_{10} = -0.0420333$	$a_{11} = 0.0347040$
$a_{12} = -0.0291807$	$a_{13} = 0.0249129$	$a_{14} = -0.0215458$	$a_{15} = 0.0188421$
$a_{16} = -0.0166380$	$a_{17} = 0.0148174$	$a_{18} = -0.0132962$	$a_{19} = 0.0120123$
$a_{20} = -0.0109189$	$a_{21} = 0.0099801$	$a_{22} = -0.0091683$	$a_{23} = 0.0084617$
$a_{24} = -0.0078430$	$a_{25} = 0.0072984$	$a_{26} = -0.0068167$	$a_{27} = 0.0063887$
$a_{28} = -0.0060069$	$a_{29} = 0.0056650$	$a_{30} = -0.0053578$	$a_{31} = 0.0050810$
$a_{32} = -0.0048308$	$a_{33} = 0.0046040$	$a_{34} = -0.0043981$	$a_{35} = 0.0042107$
$a_{36} = -0.0040398$	$a_{37} = 0.0038837$	$a_{38} = -0.0037409$	$a_{39} = 0.0036102$
$a_{40} = -0.0034903$	$a_{41} = 0.0033803$	$a_{42} = -0.0032793$	$a_{43} = 0.0031866$
$a_{44} = -0.0031015$	$a_{45} = 0.0030233$	$a_{46} = -0.0029516$	$a_{47} = 0.0028858$
$a_{48} = -0.0028256$	$a_{49} = 0.0027705$	$a_{50} = -0.0027203$	$a_{51} = 0.0026747$
$a_{52} = -0.0026333$	$a_{53} = 0.0025960$	$a_{54} = -0.0025626$	$a_{55} = 0.0025328$
$a_{56} = -0.0025066$	$a_{57} = 0.0024837$	$a_{58} = -0.0024642$	$a_{59} = 0.0024478$
$a_{60} = -0.0024345$	$a_{61} = 0.0024242$	$a_{62} = -0.0024169$	$a_{63} = 0.0024125$

8. Since $(\cos x)^2 = \frac{1}{2} + \frac{1}{2}\cos 2x$,

$$\sum_{j=0}^{2m-1} (\cos mx_j)^2 = \frac{1}{2}\sum_{j=0}^{2m-1} 1 + \frac{1}{2}\sum_{j=0}^{2m-1} \cos 2mx_j = m + \frac{1}{2}\sum_{j=0}^{2m-1} \cos 2mx_j.$$

However,

$$\cos 2mx_j = \cos 2m\left(-\pi + \frac{j}{m}\pi\right) = \cos(2j\pi - 2m\pi) = \cos(2j - 2m)\pi = (-1)^{2j-2m} = 1.$$

Thus

$$\sum_{j=0}^{2m-1} (\cos mx_j)^2 = m + \frac{1}{2}\sum_{j=0}^{2m-1} 1 = m + m = 2m.$$

9. From Eq. (8.28),

$$c_k = \sum_{j=0}^{2m-1} y_j e^{\frac{\pi ijk}{m}} = \sum_{j=0}^{2m-1} y_j (\zeta)^{jk} = \sum_{j=0}^{2m-1} y_j \left(\zeta^k\right)^j.$$

Thus

$$c_k = \left(1, \zeta^k, \zeta^{2k}, \ldots, \zeta^{(2m-1)k}\right)^t \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{2m-1} \end{bmatrix},$$

and the result follows.

10. We have $\mathbf{c} = A\mathbf{d}$, $\mathbf{d} = B\mathbf{e}$, $\mathbf{e} = C\mathbf{f}$, and $\mathbf{f} = D\mathbf{y}$, where

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & -i \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-i+1}{\sqrt{2}} & \frac{-i+1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-i-1}{\sqrt{2}} & \frac{-i-1}{\sqrt{2}} \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix},$$

and

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & \frac{i-1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i-1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-(i+1)}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(i+1)}{\sqrt{2}} \end{bmatrix}.$$

Note that $\mathbf{c} = ABCD\mathbf{y}$, which would give Eq. (8.28) if expanded.