1. The Padé approximations of degree two for $f(x) = e^{2x}$ are:

$$\begin{aligned} n &= 2, m = 0 : r_{2,0}(x) = 1 + 2x + 2x^2 \\ n &= 1, m = 1 : r_{1,1}(x) = (1 + x)/(1 - x) \\ n &= 0, m = 2 : r_{0,2}(x) = (1 - 2x + 2x^2)^{-1} \end{aligned}$$

i	x_i	$f(x_i)$	$r_{2,0}(x_i)$	$r_{1,1}(x_i)$	$r_{0,2}(x_i)$
1	0.2	1.4918	1.4800	1.5000	1.4706
2	0.4	2.2255	2.1200	2.3333	1.9231
3	0.6	3.3201	2.9200	4.0000	1.9231
4	0.8	4.9530	3.8800	9.0000	1.4706
5	1.0	7.3891	5.0000	undefined	1.0000

2. The Padé approximations of degree three for $f(x) = x\ln(x+1)$ are:

$m = 0,\ n = 3$: $x^2 - \frac{1}{2}x^3$

$m = 1,\ n = 2$: $\dfrac{x^2}{1 + \frac{1}{2}x}$

$m = 1,\ n = 2$; $m = 2,\ n = 1$; and $m = 3,\ n = 0$: $\dfrac{x^2}{1 + \frac{1}{2}x}$

i	x_i	$f(x_i)$	$r_{3,0}(x_i)$	$r_{2,1}(x_i)$
1	0.2	0.03646431	0.03600000	0.03636364
2	0.4	0.13458889	0.12800000	0.13333333
3	0.6	0.28200218	0.25200000	0.27692308
4	0.8	0.47022933	0.38400000	0.45714286
5	1.0	0.69314718	0.50000000	0.66666666

3. The Padé approximation of degree five for $f(x) = e^x$ with $n = 2$ and $m = 3$ is:

$r_{2,3}(x) = (1 + \frac{2}{5}x + \frac{1}{20}x^2)/(1 - \frac{3}{5}x + \frac{3}{20}x^2 - \frac{1}{60}x^3)$

i	x_i	$f(x_i)$	$r_{2,3}(x_i)$
1	0.2	1.22140276	1.22140277
2	0.4	1.49182470	1.49182561
3	0.6	1.82211880	1.82213210
4	0.8	2.22554093	2.22563652
5	1.0	2.71828183	2.71875000

4. The Padé approximations of degree five for $f(x) = e^x$ with $n = 3$ and $m = 2$ is:
$r_{3,2}(x) = \left(1 + \frac{3}{5}x + \frac{3}{20}x^2 + \frac{1}{60}x^3\right) / \left(1 - \frac{2}{5}x + \frac{1}{20}x^2\right)$

i	x_i	$f(x_i)$	$r_{3,2}(x_i)$
1	0.2	1.22140276	1.22140275
2	0.4	1.49182470	1.49182390
3	0.6	1.82211880	1.82210797
4	0.8	2.22554093	2.22546816
5	1.0	2.71828183	2.71794872

5. The Padé approximations of degree six for $f(x) = \sin x$ with $m = n = 3$ is:
$r_{3,3}(x) = (x - \frac{7}{60}x^3)/(1 + \frac{1}{20}x^2)$

i	x_i	$f(x_i)$	Maclaurin polynomial of degree 6	$r_{3,3}(x_i)$
0	0.0	0.00000000	0.00000000	0.00000000
1	0.1	0.09983342	0.09966675	0.09938640
2	0.2	0.19866933	0.19733600	0.19709571
3	0.3	0.29552021	0.29102025	0.29246305
4	0.4	0.38941834	0.37875200	0.38483660
5	0.5	0.47942554	0.45859375	0.47357724

6. The Padé approximations of degree six for $f(x) = \sin x$ are as follows.

(a) With $n = 2$ and $m = 4$: $r_{2,4}(x) = x/\left(1 + \frac{1}{6}x^2 + \frac{7}{360}x^4\right)$

(b) With $n = 4$ and $m = 2$: $r_{4,2}(x) = \left(x - \frac{7}{60}x^3\right) / \left(1 + \frac{1}{20}x^2\right)$

i	x_i	$f(x_i)$	$r_{2,4}(x_i)$	$r_{4,2}(x_i)$
0	0.0	0.00000000	0.00000000	0.00000000
1	0.1	0.09983342	0.09983342	0.09938640
2	0.2	0.19866933	0.19866936	0.19709571
3	0.3	0.29552021	0.29552065	0.29246305
4	0.4	0.38941834	0.38942158	0.38483660
5	0.5	0.47942554	0.47944065	0.47357724

7. The Padé approximations of degree five are:

(a) $r_{0,5}(x) = (1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5)^{-1}$

(b) $r_{1,4}(x) = (1 - \frac{1}{5}x)/(1 + \frac{4}{5}x + \frac{3}{10}x^2 + \frac{1}{15}x^3 + \frac{1}{120}x^4)$

(c) $r_{3,2}(x) = (1 - \frac{3}{5}x + \frac{3}{20}x^2 - \frac{1}{60}x^3)/(1 + \frac{2}{5}x + \frac{1}{20}x^2)$

(d) $r_{4,1}(x) = (1 - \frac{4}{5}x + \frac{3}{10}x^2 - \frac{1}{15}x^3 + \frac{1}{120}x^4)/(1 + \frac{1}{5}x)$

i	x_i	$f(x_i)$	$r_{0,5}(x_i)$	$r_{1,4}(x_i)$	$r_{2,3}(x_i)$	$r_{4,1}(x_i)$
1	0.2	0.81873075	0.81873081	0.81873074	0.81873075	0.81873077
2	0.4	0.67032005	0.67032276	0.67031942	0.67031963	0.67032099
3	0.6	0.54881164	0.54883296	0.54880635	0.54880763	0.54882143
4	0.8	0.44932896	0.44941181	0.44930678	0.44930966	0.44937931
5	1.0	0.36787944	0.36809816	0.36781609	0.36781609	0.36805556

8. The continued fraction forms of the rational functions are shown.

(a) $1 + \cfrac{4}{x - \frac{5}{4} + \cfrac{\frac{21}{16}}{x + \frac{1}{4}}}$

(b) $\cfrac{2}{x - \frac{1}{4} + \cfrac{\frac{23}{8}}{x - \frac{63}{92} - \cfrac{\frac{406}{529}}{x + \frac{33}{23}}}}$

(c) $2x - 7 + \cfrac{10}{x - \frac{3}{10} + \cfrac{\frac{469}{100}}{x + \frac{23}{10}}}$

(d) $\frac{2}{3} - \cfrac{\frac{1}{9}}{x - \frac{1}{3} + \cfrac{7}{x + \frac{9}{7} - \cfrac{\frac{325}{49}}{x - \frac{2}{7}}}}$

9. For $f(x) = e^x$ we have the following approximations.

$r_{T_{2,0}}(x) = (1.266066T_0(x) - 1.130318T_1(x) + 0.2714953T_2(x))/T_0(x)$

$r_{T_{1,1}}(x) = (0.9945705T_0(x) - 0.4569046T_1(x))/(T_0(x) + 0.48038745T_1(x))$

$r_{T_{0,2}}(x) = 0.7940220T_0(x)/(T_0(x) + 0.8778575T_1(x) + 0.1774266T_2(x))$

i	x_i	$f(x_i)$	$r_{T_{2,0}}(x_i)$	$r_{T_{1,1}}(x_i)$	$r_{T_{0,2}}(x_i)$
1	0.25	0.77880078	0.74592811	0.78595377	0.74610974
2	0.50	0.60653066	0.56515935	0.61774075	0.58807059
3	1.00	0.36787944	0.40724330	0.36319269	0.38633199

10. For $f(x) = \cos x$ we have the following approximations.
$m = 3$, $n = 0$ and $m = 2, n = 1$:

$$\frac{0.7306893 T_0(x)}{T_0(x) + 0.3003238 T_2(x)}$$

$m = 1$, $n = 2$ and $m = 0, n = 3$:

$$\frac{0.7651975 T_0(x) - 0.2298070 T_2(x)}{T_0(x)}$$

This gives

x	$f(x)$	$r_{T_{0,3}}(x)$ and $r_{T_{1,2}}(x)$	$r_{T_{2,1}}(x)$ and $r_{T_{3,0}}(x)$
0.78539816	0.70710678	0.68276861	0.71149148
1.04719755	0.50000000	0.53792021	0.49098135

11. For $f(x) = \sin x$ we have the following approximations.
$r_{T_{2,2}}(x) = \dfrac{0.91747 T_1(x)}{T_0(x) + 0.088914 T_2(x)}$

i	x_i	$f(x_i)$	$r_{T_{2,2}}(x_i)$
0	0.00	0.00000000	0.00000000
1	0.10	0.09983342	0.09093843
2	0.20	0.19866933	0.18028797
3	0.30	0.29552021	0.26808992
4	0.40	0.38941834	0.35438412

12. For $f(x) = e^x$ we have the following degree five approximations.
When $m = 5, n = 0$:

$$\frac{0.7898486T_0(x)}{T_0(x) - 0.8927799T_1(x) + 0.2144414T_2(x) - 0.03502476T_3(x) + 0.004335741T_4(x) - 0.0004335974T_5(x)}$$

When $m = 4, n = 1$:

$$\frac{0.8698859T_0(x) + 0.1792990T_1(x)}{T_0(x) - 0.7319036T_1(x) + 0.1308634T_2(x) - 0.01374200T_3(x) + 0.0007516311T_4(x)}$$

When $m = 3, n = 2$:

$$\frac{0.9455983T_0(x) + 0.3537814T_1(x) + 0.02028345T_2(x)}{T_0(x) - 0.5848039T_1(x) + 0.07467597T_2(x) - 0.004402997T_3(x)}$$

When $m = 2, n = 3$:

$$\frac{1.055167T_0(x) + 0.6127802T_1(x) + 0.07740801T_2(x) + 0.004495996T_3(x)}{T_0(x) - 0.3785111T_1(x) + 0.02224353T_2(x)}$$

When $m = 1, n = 4$:

$$\frac{1.153963T_0(x) + 0.8522588T_1(x) + 0.1549949T_2(x) + 0.01686746T_3(x) + 0.001023136T_4(x)}{T_0(x) - 0.1983568T_1(x)}$$

$m = 0, n = 5$:

$$1.266066T_0(x) + 1.130318T_1(x) + 0.2714953T_2(x) + 0.04433685T_3(x)$$

$$+0.005474240T_4(x) + 0.0005429263T_5(x)$$

i	x_i	$f(x_i)$	$r_{T_{0,5}}(x_i)$	$r_{T_{1,4}}(x_i)$	$r_{T_{2,3}}(x_i)$	$r_{T_{3,2}}(x_i)$	$r_{T_{4,1}}(x_i)$	$r_{T_{5,0}}(x_i)$
1	0.2	1.22140276	1.22137251	1.22142042	1.22140929	1.22141264	1.29573091	1.22142198
2	0.4	1.49182470	1.49190745	1.49184755	1.49184841	1.49183231	1.54914242	1.49179061
3	0.6	1.82211880	1.82224269	1.82211712	1.82213166	1.82211572	1.84678705	1.82208177
4	0.8	2.22554093	2.22539680	2.22551178	2.22550877	2.22553290	2.19970546	2.22557527
5	1.0	2.71828183	2.71856417	2.71831087	2.71832589	2.71828966	2.62151591	2.71823332

13. (a) $e^x = e^{M\ln\sqrt{10}+s} = e^{M\ln\sqrt{10}}e^s = e^{\ln 10\frac{M}{2}}e^s = 10^{\frac{M}{2}}e^s$

(b) $e^s \approx \left(1+\frac{1}{2}s+\frac{1}{10}s^2+\frac{1}{120}s^3\right) / \left(1-\frac{1}{2}s+\frac{1}{10}s^2-\frac{1}{120}s^3\right)$, with $|\text{error}| \leq 3.75\times 10^{-7}$.

(c) Set $M = \text{round}(0.8685889638x)$, $s = x - M/(0.8685889638)$, and
$\hat{f} = \left(1+\frac{1}{2}s+\frac{1}{10}s^2+\frac{1}{120}s^3\right) / \left(1-\frac{1}{2}s+\frac{1}{10}s^2-\frac{1}{120}s^3\right)$. Then $f = (3.16227766)^M\hat{f}$.

14. (a) Since

$$\sin|x| = \sin(M\pi+s) = \sin M\pi\cos s + \cos M\pi \sin s = (-1)^M \sin s,$$

we have

$$\sin x = \text{ sign } x \sin|x| = \text{sign }(x)(-1)^M \sin s.$$

(b) We have

$$\sin x \approx \left(s - \frac{31}{294}s^3\right) \Big/ \left(1+\frac{3}{49}s^2+\frac{11}{5880}s^3\right),$$

with $|\text{error}| \leq 2.84\times 10^{-4}$.

(c) Set $M = \text{Round}(|x|/\pi)$, $s = |x| - M\pi$, and
$f_1 = \left(s-\frac{31}{294}s^3\right) \Big/ \left(1+\frac{3}{49}s^2+\frac{11}{5880}s^4\right)$.
Then $f = (-1)^M f_1 \cdot x/|x|$ is the approximation.

(d) Set $y = x + \frac{\pi}{2}$, and repeat part (c) with y in place of x.