1. The interpolating polynomials of degree two are:

   (a) $P_2(x) = 2.377443 + 1.590534(x - 0.8660254) + 0.5320418(x - 0.8660254)x$

   (b) $P_2(x) = 0.7617600 + 0.8796047(x - 0.8660254)$

   (c) $P_2(x) = 1.052926 + 0.4154370(x - 0.8660254) - 0.1384262x(x - 0.8660254)$

   (d) $P_2(x) = 0.5625 + 0.649519(x - 0.8660254) + 0.75x(x - 0.8660254)$

2. The interpolating polynomials of degree three are:

   (a) $P_3(x) = 2.519044 + 1.945377(x - 0.9238795) + 0.7047420(x - 0.9238795)(x - 0.3826834)$
   $+0.1751757(x - 0.9238795)(x - 0.3826834)(x + 0.3826834)$

   (b) $P_3(x) = 0.7979459 + 0.7844380(x - 0.9238795) - 0.1464394(x - 0.9238795)(x - 0.3826834)$
   $-0.1585049(x - 0.9238795)(x - 0.3826834)(x + 0.3826834)$

   (c) $P_3(x) = 1.072911 + 0.3782067(x - 0.9238795) - 0.09799213(x - 0.9238795)(x - 0.3826834)$
   $+0.04909073(x - 0.9238795)(x - 0.3826834)(x + 0.3826834)$

   (d) $P_3(x) = 0.7285533 + 1.306563(x - 0.9238795) + 0.9999999(x - 0.9238795)(x - 0.3826834)$

3. Bounds for the maximum errors of polynomials in Exercise 1 are:

   (a) 0.1132617 (b) 0.04166667 (c) 0.08333333 (d) 1.000000

4. Bounds for the maximum errors of polynomials in Exercise 3 are:

   (a) 0.01415772 (b) 0.004382661 (c) 0.03125000 (d) 0.1250000

5. The zeros of $\tilde{T}_3$ produce the following interpolating polynomials of degree two.

   (a) $P_2(x) = 0.3489153 - 0.1744576(x - 2.866025) + 0.1538462(x - 2.866025)(x - 2)$

   (b) $P_2(x) = 0.1547375 - 0.2461152(x - 1.866025) + 0.1957273(x - 1.866025)(x - 1)$

   (c) $P_2(x) = 0.6166200 - 0.2370869(x - 0.9330127) - 0.7427732(x - 0.9330127)(x - 0.5)$

   (d) $P_2(x) = 3.0177125 + 1.883800(x - 2.866025) + 0.2584625(x - 2.866025)(x - 2)$

6. The polynomial
$$P(x) = \frac{1}{3840} + \frac{379}{384}x + \frac{637}{640}x^2 + \frac{53}{96}x^3 + \frac{43}{240}x^4$$
approximates xe^x, with error at most 0.00718.

7. The cubic polynomial $\frac{383}{384}x - \frac{5}{32}x^3$ approximates $\sin x$ with error at most 7.19×10^{-4}.

8. If $i > j$, then
$$\frac{1}{2}(T_{i+j}(x) + T_{i-j}(x)) = \frac{1}{2}(\cos(i+j)\theta + \cos(i-j)\theta) = \cos i\theta \cos j\theta = T_i(x)T_j(x).$$

9. The change of variable $x = \cos\theta$ produces

$$\int_{-1}^{1} \frac{T_n^2(x)}{\sqrt{1-x^2}}\, dx = \int_{-1}^{1} \frac{[\cos(n \arccos x)]^2}{\sqrt{1-x^2}}\, dx = \int_{0}^{\pi} (\cos(n\theta))^2\, dx = \frac{\pi}{2}.$$

10. The zeros of $T_n(x)$ are $\overline{x}_n = \cos\left(\frac{2k-1}{2n}\pi\right)$ for $k = 1, 2, \ldots, n$

The cosine function is strictly decreasing from 0 to π, with values decreasing from $\cos 0 = 1$ to $\cos\pi = -1$. So

$$-1 = \cos(\pi) < \cos\left(\frac{2n-1}{2n}\pi\right) = \overline{x}_n < \overline{x}_{n-1} < \cdots < \overline{x}_1 = \cos\left(\frac{\pi}{2n}\right) < \cos(1) = 1$$

Hence the zeros are distinct and lie in $(-1, 1)$

11. The zeros of $T_n'(x)$ are $\overline{x'}_n = \cos\left(\frac{k\pi}{n}\right)$ for $k = 1, 2, \ldots, n-1$

The cosine function is strictly decreasing from 0 to π, with values decreasing from $\cos 0 = 1$ to $\cos\pi = -1$. So

$$-1 = \cos(\pi) < \cos\left(\frac{(n-1)\pi}{n}\right) = \overline{x'}_{n-1} < \overline{x'}_{n-2} < \cdots < \overline{x'}_1 = \cos\left(\frac{\pi}{n}\right) < \cos(1) = 1$$

Hence the zeros are distinct and lie in $(-1, 1)$