1. The linear least-squares polynomial is $1.70784x + 0.89968$.

2. The least-squares polynomial of degree two is $P_2(x) = 0.4066667 + 1.154848x + 0.03484848x^2$, with $E = 1.7035$.

3. The least-squares polynomials with their errors are, respectively,
   $P_1(x) = 0.9295140 + 0.5281021x$, with 2.457×10^{-2};
   $P_2(x) = 1.011341 - 0.3256988x + 1.147330x^2$, with 9.453×10^{-4};
   $P_3(x) = 1.000440 - 0.001540986x - 0.011505675x^2 + 1.021023x^3$ with 1.112×10^{-4}.

4. The least-squares polynomials with their errors are, respectively, $0.6208950 + 1.219621x$, with $E = 2.719 \times 10^{-5}$; $0.5965807 + 1.253293x - 0.01085343x^2$, with $E = 1.801 \times 10^{-5}$; and $0.6290193 + 1.185010x + 0.03533252x^2 - 0.01004723x^3$, with $E = 1.741 \times 10^{-5}$.

5. (a) The linear least-squares polynomial is $P_1(x) = 1.665540x - 0.5124568$, with error 0.33559.
   (b) The least-squares polynomial of degree two is $P_2(x) = 1.129424x^2 - 0.3114035x + 0.08514393$, with error 2.4199×10^{-3}.
   (c) The least-squares polynomial of degree three is $P_3(x) = 0.2662081x^3 + 0.4029322x^2 + 0.2483857x - 0.01840140$, with error 5.0747×10^{-6}.
   (d) The least-squares approximation of the form be^{ax} is $f(x) = 0.04570748e^{2.707295x}$, with error 1.0750.
   (e) The least-squares approximation of the form bx^a is $f(x) = 0.9501565x^{1.872009}$, with error 0.054477.

6. (a) The linear least-squares polynomial is $72.0845x - 194.138$, with error 329.
   (b) The least-squares polynomial of degree two is $6.61821x^2 - 1.14352x + 1.23556$, with error 1.44×10^{-3}.
   (c) The least-squares polynomial of degree three is $-0.0136742x^3 + 6.84557x^2 - 2.37919x + 3.42904$, with error 5.27×10^{-4}.
   (d) The least-squares approximation of the form be^{ax} is $24.2588e^{0.372382x}$, with error 418.
   (e) The least-squares approximation of the form bx^a is $6.23903x^{2.01954}$, with error 0.00703.

7. (a) $k = 0.8996$, $E(k) = 0.295$
   (b) $k = 0.9052$, $E(k) = 0.128$ Part (b) fits the total experimental data best.

8. $P_1(x) = 0.22335x - 0.80283$. For minimal A, 406; for minimal D, 272. The prediction for an A is certainly unreasonable.

9. The least squares line for the point average is 0.101 (ACT score) + 0.487.

10. The percent occurrence is $-0.0022550x$(average weight) $+13.146$.

11. The linear least-squares polynomial gives $y \approx 0.17952x + 8.2084$.

12. The linear least-squares polynomial is $1.600393x + 25.92175$

13. (a) $\ln R = \ln 1.304 + 0.5756 \ln W$

(b) $E = 25.25$

(c) $\ln R = \ln 1.051 + 0.7006 \ln W + 0.06695(\ln W)^2$

(d) $E = \sum_{i=1}^{37} \left(R_i - bW_i^a e^{c(\ln W_i)^2} \right)^2 = 20.30$

14. For each $i = 1, \ldots, n+1$ and $j = 1, \ldots, n+1$, $a_{ij} = a_{ji} = \sum_{k=1}^{m} x_k^{i+j-2}$, so $A = (a_{ij})$ is symmetric.

Suppose A is singular and $\mathbf{c} \neq \mathbf{0}$ satisfies $\mathbf{c}^t A \mathbf{c} = \mathbf{0}$. Then

$$0 = \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} a_{ij} c_i c_j = \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} \left(\sum_{k=1}^{m} x_k^{i+j-2} \right) c_i c_j = \sum_{k=1}^{m} \left[\sum_{i=1}^{n+1} \sum_{j=1}^{n+1} c_i c_j x_k^{i+j-2} \right],$$

so

$$\sum_{k=1}^{m} \left(\sum_{i=1}^{n+1} c_i x_k^{i-1} \right)^2 = 0.$$

Define $P(x) = c_1 + c_2 x + \ldots + c_{n+1} x^n$. Then $\sum_{k=1}^{m} [P(x_k)]^2 = 0$ and $P(x)$ has roots $x_1, \ldots, x_m$. Since the roots are distinct and $m > n$, $P(x)$ must be the zero polynomial. Thus, $c_1 = c_2 = \ldots = c_{n+1} = 0$, and A must be nonsingular.