1. The l_∞ condition numbers are:

(a) 50 (b) 241.37 (c) 600,002 (d) 339,866

2. The l_∞ condition numbers are:

(a) 12.24012756 (b) 12.24012756 (c) 12 (d) 198.17

3. We have

	$\|\mathbf{x}-\hat{\mathbf{x}}\|_\infty$	$K_\infty(A)\|\mathbf{b}-A\hat{\mathbf{x}}\|_\infty/\|A\|_\infty$
(a)	8.571429×10^{-4}	1.238095×10^{-2}
(b)	0.1	3.832060
(c)	0.04	0.8
(d)	20	1.152440×10^{5}

4. We have

	$\|\mathbf{x}-\hat{\mathbf{x}}\|_\infty$	$K_\infty(A)\|\mathbf{b}-A\hat{\mathbf{x}}\|_\infty/\|A\|_\infty$
(a)	20	65.03241
(b)	0.02	720.5764
(c)	0.1	3.727412×10^{-1}
(d)	6.551700×10^{-2}	9.059201

5. Gaussian elimination and iterative refinement give the following results.

(a) (i) $(-10.0, 1.01)^t$, (ii) $(10.0, 1.00)^t$

(b) (i) $(12.0, 0.499, -1.98)^t$, (ii) $(1.00, 0.500, -1.00)^t$

(c) (i) $(0.185, 0.0103, -0.0200, -1.12)^t$, (ii) $(0.177, 0.0127, -0.0207, -1.18)^t$

(d) (i) $(0.799, -3.12, 0.151, 4.56)^t$, (ii) $(0.758, -3.00, 0.159, 4.30)^t$

6. Gaussian elimination and iterative refinement give the following results.

(a) $(12.00, 0.9990)^t$, $(10.00, 1.000)^t$

(b) $(1.200, 0.5002, -1.380)^t$, $(1.000, 0.5000, -0.9998)^t$

(c) $(0.1756, 0.01305, -0.02075, -1.192)^t$, $(0.1768, 0.01269, -0.02065, -1.182)^t$

(d) $(0.7963, -3.152, 0.1705, 4.615)^t$, $(0.7889, -3.128, 0.1678, 4.561)^t$

7. The matrix is ill-conditioned since $K_\infty = 60002$. We have $\tilde{\mathbf{x}} = (-1.0000, 2.0000)^t$.

8. The matrix A is ill-conditioned since $K_\infty(A) = 600,002$ and $\hat{\mathbf{x}} = (1.818192, 0.5909091)^t$

9. For any vector $\mathbf{x}$, we have

$$\|\mathbf{x}\| = \|A^{-1}A\mathbf{x}\| \le \|A^{-1}\| \ \|A\mathbf{x}\|, \quad \text{so} \quad \|A\mathbf{x}\| \ge \frac{\|\mathbf{x}\|}{\|A^{-1}\|}.$$

Let $\mathbf{x} \neq \mathbf{0}$ be such that $\|\mathbf{x}\| = 1$ and $B\mathbf{x} = \mathbf{0}$. Then

$$\|(A-B)\mathbf{x}\| = \|A\mathbf{x}\| \ge \frac{\|\mathbf{x}\|}{\|A^{-1}\|} \quad \text{and} \quad \frac{\|(A-B)\mathbf{x}\|}{\|A\|} \ge \frac{1}{\|A^{-1}\| \ \|A\|} = \frac{1}{K(A)}.$$

Since $\|\mathbf{x}\| = 1$,

$$\|(A-B)\mathbf{x}\| \le \|A-B\| \ \|\mathbf{x}\| = \|A-B\| \quad \text{and} \quad \frac{\|A-B\|}{\|A\|} \ge \frac{1}{K(A)}.$$

10. The approximate condition numbers are as follows:

(a) With $B = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$, we have $K_\infty(A) \ge 30{,}000$.

(b) With $B = \begin{bmatrix} 4.0 & 1.6 \\ 7.0 & 2.8 \end{bmatrix}$, we have $K_\infty(A) \ge \dfrac{97}{3}$.

11. (a) $K_\infty\left(H^{(4)}\right) = 28,375$

(b) $K_\infty\left(H^{(5)}\right) = 943,656$

(c) actual solution $\mathbf{x} = (-124, 1560, -3960, 2660)^t$;
approximate solution $\tilde{\mathbf{x}} = (-124.2,\ 1563.8, -3971.8, 2668.8)^t$; $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty = 11.8$; and $\dfrac{\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty}{\|\mathbf{x}\|_\infty} = 0.02980$;

$$\frac{K_\infty(A)}{1 - K_\infty(A)\left(\frac{\|\delta A\|_\infty}{\|A\|_\infty}\right)}\left[\frac{\|\delta b\|_\infty}{\|b\|_\infty} + \frac{\|\delta A\|_\infty}{\|A\|_\infty}\right] = \frac{28375}{1 - 28375\left(\frac{6.\overline{6}\times 10^{-6}}{2.08\overline{3}}\right)}\left[0 + \frac{6.\overline{6}\times 10^{-6}}{2.08\overline{3}}\right]$$
$$= 0.09987.$$

12. For the 3×3 Hilbert matrix H, we have

$$\hat{H}^{-1} = \begin{bmatrix} 8.968 & -35.77 & 29.77 \\ -35.77 & 190.6 & -178.6 \\ 29.77 & -178.6 & 178.6 \end{bmatrix}, \qquad \hat{H} = \begin{bmatrix} 0.9799 & 0.4870 & 0.3238 \\ 0.4860 & 0.3246 & 0.2434 \\ 0.3232 & 0.2433 & 0.1949 \end{bmatrix},$$

and $\|H - \hat{H}\|_\infty = 0.04260$.