

1. The solutions to the linear systems are as follows.

(a)  $x_1 = 11/20, x_2 = 3/10, x_3 = 2/5$

(b)  $x_1 = 176, x_2 = -50, x_3 = 24$

2. The solutions to the linear systems are as follows.

(a)  $x_1 = -3, x_2 = 3, x_3 = 1$

(b)  $x_1 = \frac{1}{2}, x_2 = -\frac{9}{2}, x_3 = \frac{7}{2}$

3. (a)  $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b)  $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c)  $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(d)  $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. (a)  $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(b)  $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c)  $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d)  $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

$$5. \quad (a) \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 4.5 & 7.5 \\ 0 & 0 & -4 \end{bmatrix}$$

$$(b) \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -2.106719 & 1 & 0 \\ 3.067193 & 1.197756 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1.012 & -2.132 & 3.104 \\ 0 & -0.3955257 & -0.4737443 \\ 0 & 0 & -8.939141 \end{bmatrix}$$

$$(c) \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 1 & -1.333333 & 2 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(d) \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.849190 & 1 & 0 & 0 \\ -0.4596433 & -0.2501219 & 1 & 0 \\ 2.768661 & -0.3079435 & -5.352283 & 1 \end{bmatrix}$$

and

$$U = \begin{bmatrix} 2.175600 & 4.023099 & -2.173199 & 5.196700 \\ 0 & 13.43947 & -4.018660 & 10.80698 \\ 0 & 0 & -0.8929510 & 5.091692 \\ 0 & 0 & 0 & 12.03614 \end{bmatrix}$$

6. (a)  $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1/2 & 1 \end{bmatrix}$  and  $U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 1/2 \end{bmatrix}$

(b)  $L = \begin{bmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 6/5 & -38/11 & 1 \end{bmatrix}$  and  $U = \begin{bmatrix} 1/3 & 1/2 & -1/4 \\ 0 & 11/30 & 21/40 \\ 0 & 0 & 241/88 \end{bmatrix}$

(c)  $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 1 & -6/7 & 1 & 0 \\ -1 & 6/7 & -4/25 & 1 \end{bmatrix}$

and

$$U = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 7/2 & 3 & 0 \\ 0 & 0 & 25/7 & 4 \\ 0 & 0 & 0 & 141/25 \end{bmatrix}$$

(d)  $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.606196 & -0.168465 & 1 & 0 \\ 0.413289 & 0.778816 & -0.707723 & 1 \end{bmatrix}$

and

$$U = \begin{bmatrix} 5.1312 & 1.414 & 3.141 & 0 \\ 0 & 5.193 & -2.197 & 5.92056 \\ 0 & 0 & 4.25195 & 4 \\ 0 & 0 & 0 & 12.6828 \end{bmatrix}$$

7. The modified  $LU$  algorithm gives the following:

(a)  $x_1 = 1, x_2 = 2, x_3 = -1$

(b)  $x_1 = 1, x_2 = 1, x_3 = 1$

(c)  $x_1 = 1.5, x_2 = 2, x_3 = -1.199998, x_4 = 3$

(d)  $x_1 = 2.939851, x_2 = 0.07067770, x_3 = 5.677735, x_4 = 4.379812$

8. The modified  $LU$  algorithm gives the following:

(a)  $x_1 = -12, x_2 = -14, x_3 = 17$

(b)  $x_1 = -495/241, x_2 = 840/241, x_3 = 56/241$

(c)  $x_1 = -29/47, x_2 = 58/47, x_3 = 32/141, x_4 = 52/141$

(d)  $x_1 = -0.706123, x_2 = -0.187410, x_3 = 0.569188, x_4 = 0.528704$

$$9. \quad (a) \quad P^tLU = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$(b) \quad P^tLU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 6 \\ 0 & 0 & 4 \end{bmatrix}$$

$$(c) \quad P^tLU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -2 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$(d) \quad P^tLU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -3 & -1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10. (a) To compute  $P^tLU$  requires  $\frac{1}{3}n^3 - \frac{1}{3}n$  Multiplications/Divisions and  $\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$  Additions/Subtractions.
- (b) If  $\tilde{P}$  is obtained from  $P$  by a simple row interchange, then  $\det \tilde{P} = -\det P$ . Thus, if  $\tilde{P}$  is obtained from  $P$  by  $k$  interchanges, we have  $\det \tilde{P} = (-1)^k \det P$ .
- (c) Only  $n - 1$  multiplications are needed in addition to the operations in part (a).
- (d) We have  $\det A = -741$ . Factoring and computing  $\det A$  requires 75 Multiplications/Divisions and 55 Additions/Subtractions.

11. (a) The steps in Algorithm 6.4 give the following:

	Multiplications/Divisions	Additions/Subtractions
Step 2	$n - 1$	0
Step 4	$\sum_{i=2}^{n-1} i - 1$	$\sum_{i=2}^{n-1} i - 1$
Step 5	$\sum_{i=2}^{n-1} \sum_{j=i+1}^n [2(i-1) + 1]$	$\sum_{i=2}^{n-1} \sum_{j=i+1}^n 2(i-1)$
Step 6	$n - 1$	$n - 1$
Totals	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$

- (b) The equations are given by

$$y_1 = \frac{b_1}{l_{11}} \quad \text{and} \quad y_i = b_i - \sum_{j=1}^{i-1} \frac{l_{ij}y_j}{l_{ii}}, \quad \text{for } i = 2, \dots, n.$$

If we assume that  $l_{ii} = 1$ , for each  $i = 1, 2, \dots, n$ , then the number of Multiplications/Divisions is

$$\sum_{i=2}^n (i-1) = \frac{n(n-1)}{2},$$

and the number of Additions/Subtractions is the same.

- (c)

	Multiplications/divisions	Additions/subtractions
Factoring into $LU$	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$
Solving $Ly = b$	$\frac{1}{2}n^2 - \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$
Solving $Ux = y$	$\frac{1}{2}n^2 + \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$
Total	$\frac{1}{3}n^3 + n^2 - \frac{1}{3}n$	$\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n$

- (d)

	Multiplications/divisions	Additions/subtractions
Factoring into $LU$	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$
Solving $Ly^{(k)} = b^{(k)}$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$
Solving $Ux^{(k)} = y^{(k)}$	$(\frac{1}{2}n^2 + \frac{1}{2}n)m$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$
Total	$\frac{1}{3}n^3 + mn^2 - \frac{1}{3}n$	$\frac{1}{3}n^3 + (m - \frac{1}{2})n^2 - (m - \frac{1}{6})n$