1. The determinants of the matrices are:

(a) 8 (b) -8 (c) 0 (d) 0

2. The determinants of the matrices are:

(a) -8 (b) 14 (c) 0 (d) 3

3. The answers are the same as in Exercise 2.

4. The answers are the same as in Exercise 1.

5. The matrix is singular when $\alpha = -\frac{3}{2}$ and when $\alpha = 2$.

6. The matrix is singular when $\alpha = 6$.

7. The system has no solutions when $\alpha = -5$

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9. When $n = 2$, $\det A = a_{11}a_{22} - a_{12}a_{21}$ requires 2 multiplications and 1 subtraction. Since

$$2!\sum_{k=1}^{1}\frac{1}{k!} = 2 \quad \text{and} \quad 2! - 1 = 1,$$

the formula holds for $n = 2$. Assume the formula is true for $n = 2, ..., m$, and let A be an $(m+1) \times (m+1)$ matrix. Then

$$\det A = \sum_{j=1}^{m+1} a_{ij}A_{ij},$$

for any i, where $1 \leq i \leq m+1$. To compute each A_{ij} requires

$$m!\sum_{k=1}^{m-1}\frac{1}{k!} \quad \text{multiplications} \quad \text{and} \quad m! - 1 \quad \text{additions/subtractions.}$$

Thus, the number of multiplications for $\det A$ is

$$(m+1)\left[m!\sum_{k=1}^{m-1}\frac{1}{k!}\right] + (m+1) = (m+1)!\left[\sum_{k=1}^{m-1}\frac{1}{k!} + \frac{1}{m!}\right] = (m+1)!\sum_{k=1}^{m}\frac{1}{k!},$$

and the number of additions/subtractions is

$$(m+1)\left[m! - 1\right] + m = (m+1)! - 1.$$

By the principle of mathematical induction, the formula is valid for any $n \geq 2$.

10. Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{and} \quad \tilde{A} = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Expanding along the third rows gives

$$\begin{aligned} \det A =& a_{31} \det \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} - a_{32} \det \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} + a_{33} \det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ =& a_{31}(a_{12}a_{23} - a_{13}a_{22}) - a_{32}(a_{11}a_{23} - a_{13}a_{21}) + a_{33}(a_{11}a_{22} - a_{12}a_{21}) \end{aligned}$$

and

$$\begin{aligned} \det \tilde{A} =& a_{31} \det \begin{bmatrix} a_{22} & a_{23} \\ a_{12} & a_{13} \end{bmatrix} - a_{32} \det \begin{bmatrix} a_{21} & a_{23} \\ a_{11} & a_{13} \end{bmatrix} + a_{33} \det \begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix} \\ =& a_{31}(a_{13}a_{22} - a_{12}a_{23}) - a_{32}(a_{13}a_{21} - a_{11}a_{23}) + a_{33}(a_{12}a_{21} - a_{11}a_{22}) = -\det A. \end{aligned}$$

The other two cases are similar.

11. The result follows from $\det AB = \det A \cdot \det B$ and Theorem 6.17.

12. (a) The solution is $x_1 = 0$, $x_2 = 10$, and $x_3 = 26$.
   (b) We have $D_1 = -1$, $D_2 = 3$, $D_3 = 7$, and $D = 0$, and there are no solutions.
   (c) We have $D_1 = D_2 = D_3 = D = 0$, and there are infinitely many solutions.
   (d) Cramer's rule requires 39 Multiplications/Divisions and 20 Additions/Subtractions.

13. (a) If D_i is the determinant of the matrix formed by replacing the ith column of A with $\mathbf{b}$ and if $D = \det A$, then

$$x_i = D_i/D, \text{ for } i = 1, \ldots, n.$$

   (b) $(n+1)!\left(\sum_{k=1}^{n-1} \frac{1}{k!}\right) + n$ multiplications/divisions;
   $(n+1)! - n - 1$ additions/subtractions.