1. The Runge-Kutta for Systems Algorithm gives the results in the following tables.

(a)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}
0.200	2.12036583	2.12500839	1.50699185	1.51158743
0.400	4.44122776	4.46511961	3.24224021	3.26598528
0.600	9.73913329	9.83235869	8.16341700	8.25629549
0.800	22.67655977	23.00263945	21.34352778	21.66887674
1.000	55.66118088	56.73748265	56.03050296	57.10536209

(b)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}
0.500	0.95671390	0.95672798	−1.08381950	−1.08383310
1.000	1.30654440	1.30655930	−0.83295364	−0.83296776
1.500	1.34416716	1.34418117	−0.56980329	−0.56981634
2.000	1.14332436	1.14333672	−0.36936318	−0.36937457

(c)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}	w_{3i}	u_{3i}
0.5	0.70787076	0.70828683	−1.24988663	−1.25056425	0.39884862	0.39815702
1.0	−0.33691753	−0.33650854	−3.01764179	−3.01945051	−0.29932294	−0.30116868
1.5	−2.41332734	−2.41345688	−5.40523279	−5.40844686	−0.92346873	−0.92675778
2.0	−5.89479008	−5.89590551	−8.70970537	−8.71450036	−1.32051165	−1.32544426

(d)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}	w_{3i}	u_{3i}
0.2	1.38165297	1.38165325	1.00800000	1.00800000	−0.61833075	−0.61833075
0.5	1.90753116	1.90753184	1.12500000	1.12500000	−0.09090565	−0.09090566
0.7	2.25503524	2.25503620	1.34300000	1.34000000	0.26343971	0.26343970
1.0	2.83211921	2.83212056	2.00000000	2.00000000	0.88212058	0.88212056

2. The Runge-Kutta for Systems Algorithm gives the results in the following tables.

(a)

t	w_1	w_2	u_1	u_2
0.2	−0.80590898	0.28590898	−0.8059123490	0.2859123490
0.4	−0.65276041	0.77276041	−0.652770464	0.772770464
0.6	−0.60003597	1.52003597	−0.600058462	1.520058462
0.8	−0.73647147	2.61647147	−0.736516212	2.616516212
1.0	−1.19444462	4.19444462	−1.194528050	4.194528050

(b)

t	w_1	w_2	u_1	u_2
0.2	−3.62420001	5.28560000	−3.624208274	5.285611032
0.6	−5.10631940	6.48842582	−5.106356400	6.488475200
1.0	−7.15475346	8.87300454	−7.154845484	8.87312731
1.4	−10.20540766	13.02054346	−10.20559990	13.02079987
1.8	−14.90857362	19.79809804	−14.90894239	19.79858986

(c)

t	w_1	w_2	w_3	u_1	u_2	u_3
0.2	2.82820001	−1.36717100	1.29430535	2.828199216	−1.367172763	1.294303950
0.4	2.34715282	−1.70213314	1.35096843	2.347150799	−1.702138886	1.350962985
0.6	1.61165492	−2.06862759	1.12031748	1.611651361	−2.068641364	1.120303523
0.8	0.68019035	−2.55506790	0.53282506	0.680185091	−2.555096786	0.532795132
1.0	−0.38623048	−3.28594071	−0.51207026	−0.386237443	−3.285996825	−0.512128574

(d)

t	w_1	w_2	w_3	u_1	u_2	u_3
0.2	5.85399925	−6.58493690	−5.51120222	6.655197738	−6.582587709	−5.507713390
0.6	13.58890681	−2.95418927	−4.51808620	14.00287757	−2.921029767	−4.45343131
1.0	41.49825592	13.88034475	26.10907731	41.57707969	14.18586490	26.71881714
1.4	134.58778600	135.26098840	267.46411360	134.5550923	137.5527145	272.0466805
1.8	443.12659480	882.03172470	1760.52619100	443.8948036	897.3731894	1791.208540

3. The Runge-Kutta for Systems Algorithm gives the results in the following tables.

(a)

t_i	w_{1i}	y_i
0.200	0.00015352	0.00015350
0.500	0.00742968	0.00743027
0.700	0.03299617	0.03299805
1.000	0.17132224	0.17132880

(b)

t_i	w_{1i}	y_i
1.200	0.96152437	0.96152583
1.500	0.77796897	0.77797237
1.700	0.59373369	0.59373830
2.000	0.27258237	0.27258872

(c)

t_i	w_{1i}	y_i
1.000	3.73162695	3.73170445
2.000	11.31424573	11.31452924
3.000	34.04395688	34.04517155

(d)

t_i	w_{1i}	w_{2i}
1.200	0.27273759	0.27273791
1.500	1.08849079	1.08849259
1.700	2.04353207	2.04353642
2.000	4.36156675	4.36157780

4. The Runge-Kutta for Systems Algorithm gives the results in the following tables.

(a)

t	w_1	w_2	$y(t)$
0.2	2.58096738	3.92714601	2.580977391
0.4	3.62954528	6.73995658	3.629577204
0.6	5.36685193	10.90938990	5.366926682
0.8	8.12969935	17.13695552	8.129852884
1.0	12.42741665	26.46948024	12.42770981

(b)

t	w_1	w_2	$y(t)$
1.2	4.77485600	4.64248546	4.774444444
1.6	7.11125088	6.91189139	7.110625000
2.0	10.25079116	8.75041113	10.25000000
2.4	14.09462248	10.45592060	14.09361111
2.8	18.60884185	12.10964620	18.60755102

(c)

t	w_1	w_2	w_3	$y(t)$
0.2	2.98086667	0.82393333	9.46726667	2.980875497
0.6	4.16962578	5.48759308	15.03205266	4.170122771
1.0	7.89009498	14.13487020	30.45672422	7.892270823
1.4	16.74506147	32.50687231	66.24043865	16.75205379
1.8	36.77072349	72.93612397	146.58698330	36.79085705

(d)

t	w_1	w_2	w_3	$y(t)$
1.2	3.73466631	9.41446279	8.04259896	3.734666667
1.4	5.78971770	11.19022983	9.67114868	5.789714286
1.6	8.23100886	13.27065559	11.11173381	8.231000000
1.8	11.11645980	15.62867678	12.45708169	11.11644444
2.0	14.50002275	18.25003892	13.75001862	14.50000000

5. To approximate the solution of the mth–order system of first–order initial–value problems

$$u_j' = f_j(t, u_1, u_2, \ldots, u_m), \quad j = 1, 2, \ldots, m, \quad \text{for} \;\; a \le t \le b, \;\; u_j(a) = \alpha_j, \;\; j = 1, 2, \ldots, m$$

at $(n+1)$ equally spaced numbers in the interval $[a, b]$;

INPUT endpoints a, b; number of equations m; integer N; initial conditions $\alpha_1, \ldots, \alpha_m$.
OUTPUT approximations $w_{i,j}$ to $u_j(t_i)$.

STEP 1 Set $h = (b-a)/N$;
STEP 2 For $j = 1, 2, \ldots, m$ set $w_{0,j} = \alpha_j$.
STEP 3 OUTPUT $(t_0, w_{0,1}, w_{0,2}, \ldots, w_{0,m})$.
STEP 4 For $i =$ $1, 2, 3$ do Steps 5–11.
STEP 5 For $j = 1, 2, \ldots, m$ set
$k_{1,j} = hf_j(t_{i-1}, w_{i-1,1}, \ldots, w_{i-1,m})$.
STEP 6 For $j = 1, 2, \ldots, m$ set
$k_{2,j} = hf_j\left(t_{i-1} + \frac{h}{2}, w_{i-1,1} + \frac{1}{2}k_{1,1}, w_{i-1,2} + \frac{1}{2}k_{1,2}, \ldots, w_{i-1,m} + \frac{1}{2}k_{1,m}\right)$.
STEP 7 For $j = 1, 2, \ldots, m$ set
$k_{3,j} = hf_j\left(t_{i-1} + \frac{h}{2}, w_{i-1,1} + \frac{1}{2}k_{2,1}, w_{i-1,2} + \frac{1}{2}k_{2,2}, \ldots, w_{i-1,m} + \frac{1}{2}k_{2,m}\right)$.
STEP 8 For $j = 1, 2, \ldots, m$ set
$k_{4,j} = hf_j(t_{i-1} + h, w_{i-1,1} + k_{3,1}, w_{i-1,2} + k_{3,2}, \ldots, w_{i-1,m} + k_{3,m})$.
STEP 9 For $j = 1, 2, \ldots, m$ set
$w_{i,j} = w_{i-1,j} + (k_{1,j} + 2k_{2,j} + 2k_{3,j} + k_{4,j})/6$.
STEP 10 Set $t_i = a + ih$.
STEP 11 OUTPUT $(t_i, w_{i,1}, w_{i,2}, \ldots, w_{i,m})$.

STEP 12 For $i = 4, \ldots, N$ do Steps 13–16.
STEP 13 Set $t_i = a + ih$.
STEP 14 For $j = 1, 2, \ldots, m$ set

$$w_{i,j}^{(0)} = w_{i-1,j} + h\Big[55f_j(t_{i-1}, w_{i-1,1}, \ldots, w_{i-1,m}) - 59f_j(t_{i-2}, w_{i-2,1}, \ldots, w_{i-2,m}) + 37f_j(t_{i-3}, w_{i-3,1}, \ldots, w_{i-3,m}) - 9f_j(t_{i-4}, w_{i-4,1}, \ldots, w_{i-4,m})\Big]\Big/24.$$

STEP 15 For $j = 1, 2, \ldots, m$ set

$$w_{i,j} = w_{i-1,j} + h\Big[9f_j\left(t_i, w_{i,1}^{(0)}, \ldots, w_{i,m}^{(0)}\right) + 19f_j(t_{i-1}, w_{i-1,1}, \ldots, w_{i-1,m}) - 5f_j(t_{i-2}, w_{i-2,1}, \ldots, w_{i-2,m}) + f_j(t_{i-3}, w_{i-3,1}, \ldots, w_{i-3,m})\Big]\Big/24.$$

STEP 16 OUTPUT $(t_i, w_{i,1}, w_{i,2}, \ldots, w_{i,m})$.
STEP 17 STOP

6. The Adams Fourth-Order Predictor-Corrector method for systems applied to the problems in Exercise 2 gives the results in the following tables.

(a)

t_i	$w_1(t_i)$	$u_1(t_i)$	$w_2(t_i)$	$u_2(t_i)$
0.2	−0.80590898	−0.80591235	0.28590898	0.28591235
0.4	−0.65276394	−0.65277046	0.77276394	0.77277046
0.6	−0.60005208	−0.60005846	1.52005208	1.52005846
0.8	−0.73651161	−0.73651621	2.61651161	2.61651621
1.0	−1.19452854	−1.19452805	4.19452854	4.19452805

(b)

t_i	$w_1(t_i)$	$u_1(t_i)$	$w_2(t_i)$	$u_2(t_i)$
0.2	−3.62420001	−3.62420827	5.28560000	5.28561103
0.8	−5.10631940	−5.10635640	6.48842583	6.48847520
1.0	−7.15480610	−7.15484549	8.87307476	8.87312731
1.4	−10.20556525	−10.20559990	13.02075362	13.02079987
1.8	−14.90892678	−14.90894239	19.79856900	19.79858986

(c)

t_i	$w_1(t_i)$	$u_1(t_i)$	$w_2(t_i)$	$u_2(t_i)$	$w_3(t_i)$	$u_3(t_i)$
0.2	2.82820001	2.82819922	−1.36717100	−1.36717276	1.29430535	1.29430395
0.4	2.34715270	2.34715080	−1.70213589	−1.70213889	1.35096690	1.35096298
0.6	1.61165288	1.61165136	−2.06863957	−2.06864136	1.12030805	1.12030352
0.8	0.68018456	0.68018509	−2.55509734	−2.55509679	0.53279799	0.53279513
1.0	−0.38624142	−0.38623744	−3.28600244	−3.28599682	−0.51213132	−0.51212857

(d)

t_i	$w_1(t_i)$	$u_1(t_i)$	$w_2(t_i)$	$u_2(t_i)$	$w_3(t_i)$	$u_3(t_i)$
0.2	5.85399925	6.65519774	−6.58493690	−6.58258771	−5.51120222	−5.50771339
0.8	13.58890681	14.00287756	−2.95418927	−2.92102977	−4.51808620	−4.45343131
1.0	41.45180376	41.57707970	13.61528979	14.18586491	25.57139128	26.71881709
1.4	134.28466702	134.55509227	132.41529876	137.55271457	261.76615143	272.04668062
1.8	441.61837376	443.89480356	860.12763808	897.37318983	1716.71350110	1791.20854020

7. The Adams Fourth-Order Predictor-Corrector method for systems applied to the problems in Exercise 1 gives the results in the following tables.

(a)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}
0.200	2.12036583	2.12500839	1.50699185	1.51158743
0.400	4.44122776	4.46511961	3.24224021	3.26598528
0.600	9.73913329	9.83235869	8.16341700	8.25629549
0.800	22.52673210	23.00263945	21.20273983	21.66887674
1.000	54.81242211	56.73748265	55.20490157	57.10536209

(b)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}
0.500	0.95675505	0.95672798	−1.08385916	−1.08383310
1.000	1.30659995	1.30655930	−0.83300571	−0.83296776
1.500	1.34420613	1.34418117	−0.56983853	−0.56981634
2.000	1.14334795	1.14333672	−0.36938396	−0.36937457

(c)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}	w_{3i}	u_{3i}
0.5	0.70787076	0.70828683	−1.24988663	−1.25056425	0.39884862	0.39815702
1.0	−0.33691753	−0.33650854	−3.01764179	−3.01945051	−0.29932294	−0.30116868
1.5	−2.41332734	−2.41345688	−5.40523279	−5.40844686	−0.92346873	−0.92675778
2.0	−5.88968402	−5.89590551	−8.72213325	−8.71450036	−1.32972524	−1.32544426

(d)

t_i	w_{1i}	u_{1i}	w_{2i}	u_{2i}	w_{3i}	u_{3i}
0.2	1.38165297	1.38165325	1.00800000	1.00800000	−0.61833075	−0.61833075
0.5	1.90752882	1.90753184	1.12500000	1.12500000	−0.09090527	−0.09090566
0.7	2.25503040	2.25503620	1.34300000	1.34300000	0.26344040	0.26343970
1.0	2.83211032	2.83212056	2.00000000	2.00000000	0.88212163	0.88212056

8. The approximations for the swinging pendulum problems are given in the tables:

(a)

t_i	θ
1.0	−0.365903
2.0	−0.0150563

(b)

t_i	θ
1.0	−0.338253
2.0	−0.0862680

9. The predicted number of prey, x_{1i}, and predators, x_{2i}, are given in the following table.

i	t_i	x_{1i}	x_{2i}
10	1.0	4393	1512
20	2.0	288	3175
30	3.0	32	2042
40	4.0	25	1258

10. The predicted number of prey, x_{1i}, and predators, x_{2i}, are given in the following table.

i	t_i	w_{1i}	w_{2i}
6	1.2	2211	11469
12	2.4	175	17492
18	3.6	2	19704

A stable solution is $x_1 = 8000$ and $x_2 = 4000$.