

1. The Runge-Kutta-Fehlberg Algorithm gives the results in the following tables.

(a)

i	t_i	w_i	h_i	y_i
1	0.2093900	0.0298184	0.2093900	0.0298337
3	0.5610469	0.4016438	0.1777496	0.4016860
5	0.8387744	1.5894061	0.1280905	1.5894600
7	1.0000000	3.2190497	0.0486737	3.2190993

(b)

i	t_i	w_i	h_i	y_i
1	2.2500000	1.4499988	0.2500000	1.4500000
2	2.5000000	1.8333332	0.2500000	1.8333333
3	2.7500000	2.1785718	0.2500000	2.1785714
4	3.0000000	2.5000005	0.2500000	2.5000000

(c)

i	t_i	w_i	h_i	y_i
1	1.2500000	2.7789299	0.2500000	2.7789294
2	1.5000000	3.6081985	0.2500000	3.6081977
3	1.7500000	4.4793288	0.2500000	4.4793276
4	2.0000000	5.3862958	0.2500000	5.3862944

(d)

i	t_i	w_i	h_i	y_i
1	0.2500000	1.3291478	0.2500000	1.3291498
2	0.5000000	1.7304857	0.2500000	1.7304898
3	0.7500000	2.0414669	0.2500000	2.0414720
4	1.0000000	2.1179750	0.2500000	2.1179795

2. The Runge-Kutta-Fehlberg Algorithm gives the results in the following tables.

(a)

i	t_i	w_i	h_i	y_i
1	1.0500000	1.1038574	0.0500000	1.1038574
2	1.1000000	1.2158864	0.0500000	1.2158863
3	1.1500000	1.3368393	0.0500000	1.3368393
4	1.2000000	1.4675697	0.0500000	1.4675696

(b)

i	t_i	w_i	h_i	y_i
1	0.2500000	0.2522868	0.2500000	0.2522868
2	0.5000000	0.5158867	0.2500000	0.5158868
3	0.7500000	0.7959445	0.2500000	0.7959446
4	1.0000000	1.0918182	0.2500000	1.0918183

(c)

i	t_i	w_i	h_i	y_i
1	1.1382206	-1.7834313	0.1382206	-1.7834282
3	1.6364797	-1.4399709	0.3071709	-1.4399551
5	2.6364797	-1.2340532	0.5000000	-1.2340298
6	3.0000000	-1.2000195	0.3635203	-1.2000000

(d)

i	t_i	w_i	h_i	y_i
1	0.5000000	0.0416667	0.5000000	0.0416667
2	1.0000000	0.3333333	0.5000000	0.3333333
3	1.5000000	1.1250000	0.5000000	1.1250000
4	2.0000000	2.6666667	0.5000000	2.6666667

3. The Runge-Kutta-Fehlberg Algorithm gives the results in the following tables.

(a)

i	t_i	w_i	h_i	y_i
1	1.1101946	1.0051237	0.1101946	1.0051237
5	1.7470584	1.1213948	0.2180472	1.1213947
7	2.3994350	1.2795396	0.3707934	1.2795395
11	4.0000000	1.6762393	0.1014853	1.6762391

(b)

i	t_i	w_i	h_i	y_i
4	1.5482238	0.7234123	0.1256486	0.7234119
7	1.8847226	1.3851234	0.1073571	1.3851226
10	2.1846024	2.1673514	0.0965027	2.1673499
16	2.6972462	4.1297939	0.0778628	4.1297904
21	3.0000000	5.8741059	0.0195070	5.8741000

(c)

i	t_i	w_i	h_i	y_i
1	0.1633541	-1.8380836	0.1633541	-1.8380836
5	0.7585763	-1.3597623	0.1266248	-1.3597624
9	1.1930325	-1.1684827	0.1048224	-1.1684830
13	1.6229351	-1.0749509	0.1107510	-1.0749511
17	2.1074733	-1.0291158	0.1288897	-1.0291161
23	3.0000000	-1.0049450	0.1264618	-1.0049452

(d)

i	t_i	w_i	h_i	y_i
1	0.3986051	0.3108201	0.3986051	0.3108199
3	0.9703970	0.2221189	0.2866710	0.2221186
5	1.5672905	0.1133085	0.3042087	0.1133082
8	2.0000000	0.0543454	0.0902302	0.0543455

4. Steps 3 and 6 must use the new equations. Step 4 must now use

$$R = \frac{1}{h} \left| -\frac{1}{160}K_1 - \frac{125}{17952}K_3 + \frac{1}{144}K_4 - \frac{12}{1955}K_5 - \frac{3}{44}K_6 + \frac{125}{11592}K_7 + \frac{43}{616}K_8 \right|,$$

and in Step 8 we must change to $\delta = 0.871(TOL/R)^{1/5}$. Repeating Exercise 3 using the Runge-Kutta-Verner method gives the results in the following tables.

(a)

i	t_i	w_i	h_i	y_i
1	1.42087564	1.05149775	0.42087564	1.05150868
3	2.28874724	1.25203709	0.50000000	1.25204675
5	3.28874724	1.50135401	0.50000000	1.50136369
7	4.00000000	1.67622922	0.21125276	1.67623914

(b)

i	t_i	w_i	h_i	y_i
1	1.27377960	0.31440170	0.27377960	0.31440111
4	1.93610139	1.50471956	0.20716801	1.50471717
7	2.48318866	3.19129592	0.17192536	3.19129017
11	3.00000000	5.87411325	0.05925262	5.87409998

(c)

i	t_i	w_i	h_i	y_i
1	0.50000000	-1.53788271	0.50000000	-1.53788284
5	1.26573379	-1.14736319	0.17746598	-1.14736283
9	1.99742532	-1.03615509	0.19229794	-1.03615478
14	3.00000000	-1.00494544	0.10525374	-1.00494525

(d)

i	t_i	w_i	h_i	y_i
1	0.50000000	0.29875168	0.50000000	0.29875178
2	1.00000000	0.21662609	0.50000000	0.21662642
4	1.74337091	0.08624885	0.27203938	0.08624932
6	2.00000000	0.05434531	0.03454832	0.05434551

5. (a) The number of infectives is $y(30) \approx 80295.7$.
(b) The limiting value for the number of infectives for this model is $\lim_{t \rightarrow \infty} y(t) = 100,000$.
6. With $TOL = 0.01$, $HMIN = 0.01$, $HMAX = 1$, we have

$$z(30) \approx 81, \quad x(30) \approx 19754, \quad \text{and} \quad y(30) \approx 80165.$$