1. The Modified Euler method gives the approximations in the following tables.

(a)

t	Modified Euler	$y(t)$
0.5	0.5602111	0.2836165
1.0	5.3014898	3.2190993

(b)

t	Modified Euler	$y(t)$
2.5	1.8125000	1.8333333
3.0	2.4815531	2.5000000

(c)

t	Modified Euler	$y(t)$
1.25	2.7750000	2.7789294
1.50	3.6008333	3.6081977
1.75	4.4688294	4.4793276
2.00	5.3728586	5.3862944

(d)

t	Modified Euler	$y(t)$
0.25	1.3199027	1.3291498
0.50	1.7070300	1.7304898
0.75	2.0053560	2.0414720
1.00	2.0770789	2.1179795

2. The Modified Euler method gives the approximations in the following tables.

(a)

t_i	w_i	$y(t_i)$
0.50	1.2181261	1.2140231
1.00	1.4975545	1.4898801

(b)

t_i	w_i	$y(t_i)$
1.50	2.3541667	2.3541020
2.00	2.7417451	2.7416574

(c)

t_i	w_i	$y(t_i)$
2.25	2.2454995	2.2441211
2.50	2.5671560	2.5644519
2.75	2.9691945	2.9651938
3.00	3.4565684	3.4512866

(d)

t_i	w_i	$y(t_i)$
1.25	1.4160751	1.4031990
1.50	1.0310111	1.0164101
1.75	0.75226668	0.73800977
2.00	0.54324500	0.52968710

3. The Modified Euler method gives the approximations in the following tables.

(a)

t_i	Modified Euler w_i	$y(t_i)$
1.2	1.0147137	1.0149523
1.5	1.0669093	1.0672624
1.7	1.1102751	1.1106551
2.0	1.1808345	1.1812322

(b)

t_i	Modified Euler w_i	$y(t_i)$
1.4	0.4850495	0.4896817
2.0	1.6384229	1.6612818
2.4	2.8250651	2.8765514
3.0	5.7075699	5.8741000

(c)

t_i	Modified Euler w_i	$y(t_i)$
0.4	−1.6229206	−1.6200510
1.0	−1.2442903	−1.2384058
1.4	−1.1200763	−1.1146484
2.0	−1.0391938	−1.0359724

(d)

t_i	Modified Euler w_i	$y(t_i)$
0.2	0.1742708	0.1626265
0.5	0.2878200	0.2773617
0.7	0.5088359	0.5000658
1.0	1.0096377	1.0022460

4. The Modified Euler method gives the approximations in the following tables.

(a)

t_i	Modified Euler w_i	$y(t_i)$
0.5	1.597265955	1.6
0.6	1.615015699	1.617647059
0.9	1.545108042	1.546961326
1.0	1.498430678	1.5

(b)

t_i	Modified Euler w_i	$y(t_i)$
1.1	−1.347996027	−1.347822707
1.2	−1.268565970	−1.268299404
1.9	−0.9395411781	−0.9392222368
2.0	−0.9105471247	−0.9102392264

(c)

t_i	Modified Euler w_i	$y(t_i)$
1.2	−1.72	−1.714285714
1.4	−1.561272503	−1.555555556
2.8	−1.219717333	−1.217391304
3.0	−1.202119310	−1.2

(d)

t_i	Modified Euler w_i	$y(t_i)$
0.5	1.289770701	1.289805276
0.6	1.380583709	1.380931216
0.9	1.631230851	1.632613182
1.0	1.700210296	1.701870053

5. The Midpoint method gives the approximations in the following tables.

(a)

t	Midpoint	$y(t)$
0.5	0.2646250	0.2836165
1.0	3.1300023	3.2190993

(b)

t	Midpoint	$y(t)$
2.5	1.7812500	1.8333333
3.0	2.4550638	2.5000000

(c)

t	Midpoint	$y(t)$
1.25	2.7777778	2.7789294
1.50	3.6060606	3.6081977
1.75	4.4763015	4.4793276
2.00	5.3824398	5.3862944

(d)

t	Midpoint	$y(t)$
0.25	1.3337962	1.3291498
0.50	1.7422854	1.7304898
0.75	2.0596374	2.0414720
1.00	2.1385560	2.1179795

6. The Midpoint method gives the approximations in the following tables.

(a)

t_i	w_i	$y(t_i)$
0.50	1.2154305	1.2140231
1.00	1.4932390	1.4898801

(b)

t_i	w_i	$y(t_i)$
1.50	2.3552632	2.3541020
2.00	2.7435126	2.7416574

(c)

t_i	w_i	$y(t_i)$
2.25	2.2446171	2.2441211
2.50	2.562980	2.5644519
2.75	2.9663178	2.9651938
3.00	3.4526648	3.4512866

(d)

t_i	w_i	$y(t_i)$
1.25	1.4365103	1.4031990
1.50	1.0516739	1.0164101
1.75	0.76935826	0.73800977
2.00	0.55667863	0.52968710

7. The Midpoint method gives the approximations in the following tables.

(a)

t_i	Midpoint w_i	$y(t_i)$
1.2	1.0153257	1.0149523
1.5	1.0677427	1.0672624
1.7	1.1111478	1.1106551
2.0	1.1817275	1.1812322

(b)

t_i	Midpoint w_i	$y(t_i)$
1.4	0.4861770	0.4896817
2.0	1.6438889	1.6612818
2.4	2.8364357	2.8765514
3.0	5.7386475	5.8741000

(c)

t_i	Midpoint w_i	$y(t_i)$
0.4	−1.6192966	−1.6200510
1.0	−1.2402470	−1.2384058
1.4	−1.1175165	−1.1146484
2.0	−1.0382227	−1.0359724

(d)

t_i	Midpoint w_i	$y(t_i)$
0.2	0.1722396	0.1626265
0.5	0.2848046	0.2773617
0.7	0.5056268	0.5000658
1.0	1.0063347	1.0022460

8. The Midpoint method gives the approximations in the following tables.

(a)

t_i	Midpoint w_i	$y(t_i)$
0.5	1.599403030	1.6
0.6	1.616526285	1.617647059
0.9	1.544954509	1.546961326
1.0	1.497941308	1.5

(b)

t_i	Midpoint w_i	$y(t_i)$
1.1	−1.348356626	−1.347822707
1.2	−1.269137742	−1.268299404
1.9	−0.9403364427	−0.9392222368
2.0	−0.9113264950	−0.9102392264

(c)

t_i	Midpoint w_i	$y(t_i)$
1.2	−1.738181818	−1.714285714
1.4	−1.579759806	−1.555555556
2.8	−1.227670824	−1.217391304
3.0	−1.209389666	−1.2

(d)

t_i	Midpoint w_i	$y(t_i)$
0.5	1.291506468	1.289805276
0.6	1.382581697	1.380931216
0.9	1.633368805	1.632613182
1.0	1.702247783	1.701870053

9. Heun's method gives the approximations in the following tables.

(a)

t_i	Heun w_i	$y(t_i)$
0.50	0.2710885	0.2836165
1.00	3.1327255	3.2190993

(b)

t_i	Heun w_i	$y(t_i)$
2.50	1.8464828	1.8333333
3.00	2.5094123	2.5000000

(c)

t_i	Heun w_i	$y(t_i)$
1.25	2.7788462	2.7789294
1.50	3.6080529	3.6081977
1.75	4.4791319	4.4793276
2.00	5.3860533	5.3862944

(d)

t_i	Heun w_i	$y(t_i)$
0.25	1.3295717	1.3291498
0.50	1.7310350	1.7304898
0.75	2.0417476	2.0414720
1.00	2.1176975	2.1179795

10. Heun's method gives the approximations in the following tables.

(a)

t_i	w_i	$y(t_i)$
0.50	1.2139924	1.2140231
1.00	1.4897542	1.4898801

(b)

t_i	w_i	$y(t_i)$
1.50	2.3540516	2.3541020
2.00	2.7415759	2.7416574

(c)

t_i	w_i	$y(t_i)$
2.25	2.2441050	2.2441211
2.50	2.5644270	2.5644520
2.75	2.9651642	2.9651938
3.00	3.4512546	3.4512866

(d)

t_i	w_i	$y(t_i)$
1.25	1.3981322	1.4031990
1.50	1.0114609	1.0164101
1.75	0.7338609	0.7380098
2.00	0.5262880	0.5296871

11. Heun's method gives the approximations in the following tables.

(a)

t_i	Heun w_i	$y(t_i)$
1.2	1.0149305	1.0149523
1.5	1.0672363	1.0672624
1.7	1.1106289	1.1106551
2.0	1.1812064	1.1812322

(b)

t_i	Heun w_i	$y(t_i)$
1.4	0.4895074	0.4896817
2.0	1.6602954	1.6612818
2.4	2.8741491	2.8765514
3.0	5.8652189	5.8741000

(c)

t_i	Heun w_i	$y(t_i)$
0.4	−1.6201023	−1.6200510
1.0	−1.2383500	−1.2384058
1.4	−1.1144745	−1.1146484
2.0	−1.0357989	−1.0359724

(d)

t_i	Heun w_i	$y(t_i)$
0.2	0.1614497	0.1626265
0.5	0.2765100	0.2773617
0.7	0.4994538	0.5000658
1.0	1.0018114	1.0022460

12. Heun's method gives the approximations in the following tables.

(a)

t_i	Heun w_i	$y(t_i)$
0.5	1.599939902	1.6
0.6	1.617600330	1.617647059
0.9	1.546961530	1.546961326
1.0	1.500010266	1.5

(b)

t_i	Heun w_i	$y(t_i)$
1.1	−1.347797247	−1.347822707
1.2	−1.268261121	−1.268299404
1.9	−0.9391794862	−0.9392222368
2.0	−0.9101980983	−0.9102392264

(c)

t_i	Heun w_i	$y(t_i)$
1.2	−1.710175817	−1.714285714
1.4	−1.551807512	−1.555555556
2.8	−1.216030469	−1.217391304
3.0	−1.198763172	−1.2

(d)

t_i	Heun w_i	$y(t_i)$
0.5	1.289772720	1.289805276
0.6	1.380888251	1.380931216
0.9	1.632584856	1.632613182
1.0	1.701855837	1.701870053

13. The Runge-Kutta method of order four gives the approximations in the following tables.

(a)

t_i	Runge-Kutta w_i	$y(t_i)$
0.5	0.2969975	0.2836165
1.0	3.3143118	3.2190993

(b)

t_i	Runge-Kutta w_i	$y(t_i)$
2.5	1.8333234	1.8333333
3.0	2.4999712	2.5000000

(c)

t_i	Runge-Kutta w_i	$y(t_i)$
1.25	2.7789095	2.7789294
1.50	3.6081647	3.6081977
1.75	4.4792846	4.4793276
2.00	5.3862426	5.3862944

(d)

t_i	Runge-Kutta w_i	$y(t_i)$
0.25	1.3291650	1.3291498
0.50	1.7305336	1.7304898
0.75	2.0415436	2.0414720
1.00	2.1180636	2.1179795

14. The Runge-Kutta method of order four gives the approximations in the following tables.

(a)

t_i	w_i	$y(t_i)$
0.50	1.2140409	1.2140231
1.00	1.4899213	1.4898801

(b)

t_i	w_i	$y(t_i)$
1.50	2.3541032	2.3541020
2.00	2.7416591	2.7416574

(c)

t_i	w_i	$y(t_i)$
2.25	2.2441194	2.2441211
2.50	2.5644488	2.5644519
2.75	2.9651894	2.9651938
3.00	3.4512811	3.4512866

(d)

t_i	w_i	$y(t_i)$
1.25	1.4033566	1.4031990
1.50	1.0165586	1.0164101
1.75	0.73813168	0.73800977
2.00	0.52978556	0.52968710

15. The Runge-Kutta method of order four gives the approximations in the following tables.

(a)

t_i	Runge-Kutta w_i	$y(t_i)$
1.2	1.0149520	1.0149523
1.5	1.0672620	1.0672624
1.7	1.1106547	1.1106551
2.0	1.1812319	1.1812322

(b)

t_i	Runge-Kutta w_i	$y(t_i)$
1.4	0.4896842	0.4896817
2.0	1.6612651	1.6612818
2.4	2.8764941	2.8765514
3.0	5.8738386	5.8741000

(c)

t_i	Runge-Kutta w_i	$y(t_i)$
0.4	−1.6200576	−1.6200510
1.0	−1.2384307	−1.2384058
1.4	−1.1146769	−1.1146484
2.0	−1.0359922	−1.0359724

(d)

t_i	Runge-Kutta w_i	$y(t_i)$
0.2	0.1627655	0.1626265
0.5	0.2774767	0.2773617
0.7	0.5001579	0.5000658
1.0	1.0023207	1.0022460

16. The Runge-Kutta method gives the approximations in the following tables.

(a)

t_i	Runge-Kutta w_i	$y(t_i)$
0.5	1.599998664	1.6
0.6	1.617645445	1.617647059
0.9	1.546959536	1.546961326
1.0	1.499998299	1.5

(b)

t_i	Runge-Kutta w_i	$y(t_i)$
1.1	−1.347822676	−1.347822707
1.2	−1.268299357	−1.268299404
1.9	−0.9392221816	−0.9392222368
2.0	−0.9102391733	−0.9102392264

(c)

t_i	Runge-Kutta w_i	$y(t_i)$
1.2	−1.714245180	−1.714285714
1.4	−1.555522884	−1.555555556
2.8	−1.217380872	−1.217391304
3.0	−1.199990539	−1.2

(d)

t_i	Runge-Kutta w_i	$y(t_i)$
0.5	1.289807149	1.289805276
0.6	1.380932547	1.380931216
0.9	1.632611867	1.632613182
1.0	1.701867708	1.701870053

17. Linear interpolation gives the following results.

(a) $1.0221167 \approx y(1.25) = 1.0219569,\ \ 1.1640347 \approx y(1.93) = 1.1643901$
(b) $1.9086500 \approx y(2.1) = 1.9249616,\ \ 4.3105913 \approx y(2.75) = 4.3941697$
(c) $-1.1461434 \approx y(1.3) = -1.1382768,\ \ -1.0454854 \approx y(1.93) = -1.0412665$
(d) $0.3271470 \approx y(0.54) = 0.3140018,\ \ 0.8967073 \approx y(0.94) = 0.8866318$

18. Linear interpolation gives the following results.

(a) $1.604365853 \approx y(0.54) = 1.610405698,\ \ 1.526437096 \approx y(0.94) = 1.528987046$
(b) $-1.234746062 \approx y(1.25) = -1.233151731,\ \ -0.9308429621 \approx y(1.93) = -0.9302304614$
(c) $-1.640636252 \approx y(1.3) = -1.625,\ \ -1.208278618 \approx y(2.93) = -1.205761317$
(d) $1.326095904 \approx y(0.54) = 1.326196852,\ \ 1.658822629 \approx y(0.94) = 1.661361751$

19. Linear interpolation gives the following results.

(a) $1.0227863 \approx y(1.25) = 1.0219569,\ \ 1.1649247 \approx y(1.93) = 1.1643901$
(b) $1.9153749 \approx y(2.1) = 1.9249616,\ \ 4.3312939 \approx y(2.75) = 4.3941697$
(c) $-1.1432070 \approx y(1.3) = -1.1382768,\ \ -1.0443743 \approx y(1.93) = -1.0412665$
(d) $0.3240839 \approx y(0.54) = 0.3140018,\ \ 0.8934152 \approx y(0.94) = 0.8866318$

20. Linear interpolation gives the following results.

(a) $1.6052523 \approx y(0.54) = 1.610405698,\ \ 1.5261492 \approx y(0.94) = 1.528987046$
(b) $-1.2353793 \approx y(1.25) = -1.233151731,\ \ -0.93163346 \approx y(1.93) = -0.9302304614$
(c) $-1.6589708 \approx y(1.3) = -1.625,\ \ -1.2157881 \approx y(2.93) = -1.205761317$
(d) $1.3279366 \approx y(0.54) = 1.326196852,\ \ 1.6609204 \approx y(0.94) = 1.661361751$

21. Linear interpolation gives the following results.

(a) $1.022359850 \approx y(1.25) = 1.0219569,\ \ 1.164403706 \approx y(1.93) = 1.1643901$
(b) $1.880848049 \approx y(2.1) = 1.9249616,\ \ 4.408426115 \approx y(2.75) = 4.3941697$
(c) $-1.140346956 \approx y(1.3) = -1.1382768,\ \ -1.041820256 \approx y(1.93) = -1.0412665$
(d) $0.3162569897 \approx y(0.54) = 0.3140018,\ \ 0.8886613426 \approx y(0.94) = 0.8866318$

22. Linear interpolation gives the following results.

(a) $1.607004073 \approx y(0.54) = 1.610405698,\ \ 1.528181024 \approx y(0.94) = 1.528987046$
(b) $-1.234413929 \approx y(1.25) = -1.233151731,\ \ -.9304850698 \approx y(1.93) = -0.9302304614$
(c) $-1.630991664 \approx y(1.3) = -1.625,\ \ -1.204806726 \approx y(2.93) = -1.205761317$
(d) $1.326218932 \approx y(0.54) = 1.326196852,\ \ 1.660293248 \approx y(0.94) = 1.661361751$

23. Linear interpolation gives the following results.

(a) $1.0223826 \approx y(1.25) = 1.0219569, \;\; 1.1644292 \approx y(1.93) = 1.1643901$

(b) $1.9373672 \approx y(2.1) = 1.9249616, \;\; 4.4134745 \approx y(2.75) = 4.3941697$

(c) $-1.1405252 \approx y(1.3) = -1.1382768, \;\; -1.0420211 \approx y(1.93) = -1.0412665$

(d) $0.31716526 \approx y(0.54) = 0.3140018, \;\; 0.88919730 \approx y(0.94) = 0.8866318$

24. Linear interpolation gives the following results.

(a) $1.607057376 \approx y(0.54) = 1.610405698, \;\; 1.528175041 \approx y(0.94) = 1.528987046$

(b) $-1.234455238 \approx y(1.25) = -1.233151731, \;\; -0.9305272791 \approx y(1.93) = -0.9302304614$

(c) $-1.634884032 \approx y(1.3) = -1.625, \;\; -1.206077156 \approx y(2.93) = -1.205761317$

(d) $1.326257308 \approx y(0.54) = 1.326196852, \;\; 1.660314203 \approx y(0.94) = 1.661361751$

25. Cubic Hermite interpolation gives the following results.

(a) $1.0219569 = y(1.25) \approx 1.0219550, 1.1643902 = y(1.93) \approx 1.1643898$

(b) $1.9249617 = y(2.10) \approx 1.9249217, 4.3941697 = y(2.75) \approx 4.3939943$

(c) $-1.138268 = y(1.3) \approx -1.1383036, -1.0412666 = y(1.93) \approx -1.0412862$

(d) $0.31400184 = y(0.54) \approx 0.31410579, 0.88663176 = y(0.94) \approx 0.88670653$

26. Cubic Hermite interpolation gives the following results.

(a) $1.610403108 \approx y(0.54) = 1.610405698, \;\; 1.528987622 \approx y(0.94) = 1.528987046$

(b) $-1.233149620 \approx y(1.25) = -1.233151731, \;\; -0.9302302474 \approx y(1.93) = -0.9302304614$

(c) $-1.624806746 \approx y(1.3) = -1.625, \;\; -1.214642601 \approx y(2.93) = -1.205761317$

(d) $1.326195390 \approx y(0.54) = 1.326196852, \;\; 1.661358558 \approx y(0.94) = 1.661361751$

27. The Modified Euler method and the Midpoint method give the same results when applied to the initial-value problem

$$y' = f(t,y) = -y + t + 1, \quad \text{for} \quad 0 \le t \le 1 \quad \text{and} \quad y(0) = 1$$

with any choice of h because the equation is linear in both y and t.
The equation for the Midpoint method has $w_0 = 1$ and

$$\begin{aligned} w_{i+1} &= w_i + hf(t_i + 0.5h, w_i + 0.5hf(t_i, w_i)) \\ &= w_i + hf(t_i + 0.5h, w_i + 0.5h(-w_i + t_i + 1)) \\ &= w_i + h(-w_i - 0.5h(-w_i + t_i + 1) + t_i + 0.5h + 1) \\ &= w_i(1 - h + 0.5h^2) + t_i(h - 0.5h^2) + h. \end{aligned}$$

The equation for the Modified Euler method has $w_0 = 1$ and

$$\begin{aligned} w_{i+1} &= w_i + 0.5h(f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i)) \\ &= w_i + 0.5h(-w_i + t_i + 1 - w_i - h(-w_i + t_i + 1) + t_i + h + 1) \\ &= w_i + 0.5h(-w_i + t_i + 1 - w_i - h(-w_i + t_i + 1) + t_i + h + 1) \\ &= w_i(1 - h + 0.5h^2) + t_i(h - 0.5h^2) + h. \end{aligned}$$

28. (a) The water level is 6.526747 ft.
(b) The tank will be empty in 25 min.

29. In 0.2 seconds we have approximately 2099 units of KOH.

30. Using the notation $y_{i+1} = y(t_{i+1})$, $y_i = y(t_i)$, and $f_i = f(t_i, y(t_i))$, we have

$$h\tau_{i+1} = y_{i+1} - y_i - a_1 f_i - a_2 f(t_i + \alpha_2, y_i + \delta_2 f_i).$$

Expanding y_{i+1} and $f(t_i + \alpha_2, y_i + \delta_2 f_i)$ in Taylor series about t_i and $f(t_i, y_i)$ gives

$$\begin{aligned}
h\tau_{i+1} &= (h - a_1 - a_2)f_i + \frac{h^2}{2}f_i' - a_2\alpha_2 f_t(t_i, y_i) \\
&\quad - a_2\delta_2 f_i f_y(t_i, y_i) + \frac{h^3}{6}f_i'' - a_2\frac{\alpha_2^2}{2}f_{tt}(t_i, y_i) \\
&\quad - a_2\alpha_2\delta_2 f_i f_{ty}(t_i, y_i) - a_2\frac{\delta_2^2}{2}f_i^2 f_{yy}(t_i, y_i) + \cdots \\
&= (h - a_1 - a_2)f_i + \left(\frac{h^2}{2} - a_2\alpha_2\right)f_t(t_i, y_i) \\
&\quad + \left(\frac{h^2}{2} - a_2\delta_2\right)f_i f_y(t_i, y_i) + \left(\frac{h^3}{6} - a_2\frac{\alpha_2^2}{2}\right)f_{tt}(t_i, y_i) \\
&\quad + \left(\frac{h^3}{3} - a_2\alpha_2\delta_2\right)f_i f_{ty}(t_i, y_i) + \left(\frac{h^3}{6} - a_2\frac{\delta_2^2}{2}\right)f_i^2 f_{tt}(t_i, y_i) \\
&\quad + \frac{h^3}{6}(f_t(t_i, y_i)f_y(t_i, y_i) + f_i f_y^2(t_i, y_i)) + \cdots.
\end{aligned}$$

Regardless of the choice of a_1, a_2, α_2, and δ_2, the term

$$\frac{h^3}{6}\left[f_t(y_i, t_i)f_y(t_i, y_i) + f_i f_y^2(t_i, y_i)\right]$$

cannot be canceled.

31. The appropriate constants are

$$\alpha_1 = \delta_1 = \alpha_2 = \delta_2 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \frac{1}{2} \quad \text{and} \quad \alpha_3 = \delta_3 = 1.$$