1. Euler's method gives the approximations in the following tables.

(a)

i	t_i	w_i	$y(t_i)$
1	0.500	0.0000000	0.2836165
2	1.000	1.1204223	3.2190993

(b)

i	t_i	w_i	$y(t_i)$
1	2.500	2.0000000	1.8333333
2	3.000	2.6250000	2.5000000

(c)

i	t_i	w_i	$y(t_i)$
1	1.250	2.7500000	2.7789294
2	1.500	3.5500000	3.6081977
3	1.750	4.3916667	4.4793276
4	2.000	5.2690476	5.3862944

(d)

i	t_i	w_i	$y(t_i)$
1	0.250	1.2500000	1.3291498
2	0.500	1.6398053	1.7304898
3	0.750	2.0242547	2.0414720
4	1.000	2.2364573	2.1179795

2. Euler's method gives the approximations in the following tables.

(a)

t_i	w_i
0.5	1.183939721
1.0	1.436252216

(b)

t_i	w_i
1.5	2.333333333
2.0	2.708333333

(c)

t_i	w_i
2.25	2.207106781
2.50	2.490998908
2.75	2.854680348
3.00	3.302596464

(d)

t_i	w_i
1.25	1.227324357
1.50	0.8321501572
1.75	0.5704467722
2.00	0.3788266146

3. The actual errors and error bounds for the approximations in Exercise 1 are given in the following tables.

(a)

t	Actual error	Error bound
0.5	0.2836165	11.3938
1.0	2.0986771	42.3654

(b)

t	Actual error	Error bound
2.5	0.166667	0.429570
3.0	0.125000	1.59726

(c)

t	Actual error	Error bound
1.25	0.0289294	0.0355032
1.50	0.0581977	0.0810902
1.75	0.0876610	0.139625
2.00	0.117247	0.214785

(d)

t	Actual error
0.25	0.0791498
0.50	0.0906844
0.75	0.0172174
1.00	0.118478

For Part (d), error bound formula (5.10) cannot be applied since $L = 0$.

4. The actual errors and error bounds for the approximations in Exercise 1 are given in the following tables.

(a)

t_i	$\lvert w_i - y(t_i)\rvert$	Error bound
0.5	0.030083340	0.06651369683
1.0	0.053627909	0.3254413801

(b)

t_i	$\lvert w_i - y(t_i)\rvert$	Error bound
1.5	0.020768633	0.02518894623
2.0	0.033324054	0.05494617010

(c)

t_i	$\lvert w_i - y(t_i)\rvert$	Error bound
2.25	0.037014328	0.04985076072
2.50	0.073453039	0.1100812516
2.75	0.110513485	0.1828526997
3.00	0.148690185	0.2707763338

(d)

t_i	$\lvert w_i - y(t_i)\rvert$	Error bound
1.25	0.175874613	0.3053253946
1.50	0.1842599898	0.8087218666
1.75	0.1675629993	1.638682338
2.00	0.1508604834	3.007055822

5. Euler's method gives the approximations in the following tables.

(a)

i	t_i	w_i	$y(t_i)$
2	1.200	1.0082645	1.0149523
4	1.400	1.0385147	1.0475339
6	1.600	1.0784611	1.0884327
8	1.800	1.1232621	1.1336536
10	2.000	1.1706516	1.1812322

(b)

i	t_i	w_i	$y(t_i)$
2	1.400	0.4388889	0.4896817
4	1.800	1.0520380	1.1994386
6	2.200	1.8842608	2.2135018
8	2.600	3.0028372	3.6784753
10	3.000	4.5142774	5.8741000

(c)

i	t_i	w_i	$y(t_i)$
2	0.400	−1.6080000	−1.6200510
4	0.800	−1.3017370	−1.3359632
6	1.200	−1.1274909	−1.1663454
8	1.600	−1.0491191	−1.0783314
10	2.000	−1.0181518	−1.0359724

(d)

i	t_i	w_i	$y(t_i)$
2	0.2	0.1083333	0.1626265
4	0.4	0.1620833	0.2051118
6	0.6	0.3455208	0.3765957
8	0.8	0.6213802	0.6461052
10	1.0	0.9803451	1.0022460

6. Euler's method gives the approximations in the following tables.

(a)

i	t_i	w_i
2	0.2	1.374257426
3	0.3	1.513709064
9	0.9	1.631412128
10	1.0	1.579669485

(b)

i	t_i	w_i
2	1.2	−1.253297013
3	1.3	−1.181899131
9	1.9	−0.9150285539
10	2.0	−0.8861569244

(c)

i	t_i	w_i
5	2.0	−1.248872291
6	2.2	−1.217791320
8	2.6	−1.174414016
9	2.8	−1.158657534

(d)

i	t_i	w_i
5	0.5	1.255609618
6	0.6	1.352114314
9	0.9	1.624904878
10	1.0	1.700214869

7. The actual errors for the approximations in Exercise 3 are in the following tables.

(a)

t	Actual error
1.2	0.0066879
1.5	0.0095942
1.7	0.0102229
2.0	0.0105806

(b)

t	Actual error
1.4	0.0507928
2.0	0.2240306
2.4	0.4742818
3.0	1.3598226

(c)

t	Actual error
0.4	0.0120510
1.0	0.0391546
1.4	0.0349030
2.0	0.0178206

(d)

t	Actual error
0.2	0.0542931
0.5	0.0363200
0.7	0.0273054
1.0	0.0219009

8. The actual errors for the approximations in Exercise 5 are in the following tables.

(a)

t	Actual error
0.2	0.028103580
0.3	0.045819156
0.9	0.084450802
1.0	0.079669485

(b)

t	Actual error
1.2	0.015002391
1.3	0.018712043
1.9	0.0241936829
2.0	0.0240823020

(c)

t	Actual error
2.0	0.084461042
2.2	0.076326327
2.6	0.063681222
2.8	0.058733770

(d)

t	Actual error
0.5	0.034195658
0.6	0.028816902
0.9	0.007708304
1.0	0.001655184

9. Euler's method gives the approximations in the following table.

(a)

i	t_i	w_i	$y(t_i)$
1	1.1	0.271828	0.345920
5	1.5	3.18744	3.96767
6	1.6	4.62080	5.70296
9	1.9	11.7480	14.3231
10	2.0	15.3982	18.6831

(b) Linear interpolation gives the approximations in the following table.

t	Approximation	$y(t)$	Error
1.04	0.108731	0.119986	0.01126
1.55	3.90412	4.78864	0.8845
1.97	14.3031	17.2793	2.976

(c) $h < 0.00064$

10. (a) We have

i	t_i	w_i	$y(t_i)$
1	1.05	−0.9500000	−0.9523810
2	1.10	−0.9045353	−0.9090909
11	1.55	−0.6263495	−0.6451613
12	1.60	−0.6049486	−0.6250000
19	1.95	−0.4850416	−0.5128205
20	2.00	−0.4712186	−0.5000000

(b) (i) $y(1.052) \approx -0.9481814$ (ii) $y(1.555) \approx -0.6242094$ (iii) $y(1.978) \approx -0.4773007$

(c) $h < 0.029$

11. (a) Euler's method produces the following approximation to $y(5) = 5.00674$.

	$h = 0.2$	$h = 0.1$	$h = 0.05$
w_N	5.00377	5.00515	5.00592

(b) $h = \sqrt{2 \times 10^{-6}} \approx 0.0014142$.

12. For $t > 0$, the approximations are zero and are not adequate approximations until the solution becomes close to zero. This behavior does not violate Theorem 5.9.

13. (a) $1.021957 = y(1.25) \approx 1.014978$, $1.164390 = y(1.93) \approx 1.153902$
    (b) $1.924962 = y(2.1) \approx 1.660756$, $4.394170 = y(2.75) \approx 3.526160$
    (c) $-1.138277 = y(1.3) \approx -1.103618$, $-1.041267 = y(1.93) \approx -1.022283$
    (d) $0.3140018 = y(0.54) \approx 0.2828333$, $0.8866318 = y(0.94) \approx 0.8665521$

14. (a) $1.411764706 = y(0.25) \approx 1.443983245$, $1.533594295 = y(0.93) \approx 1.615889335$
    (b) $-1.233151731 = y(1.25) \approx -1.217598072$, $-0.9302304614 = y(1.93) \approx -0.9063670650$
    (c) $-1.3125 = y(2.10) \approx, -1.233331806$ $-1.2222222222 = y(2.75) \approx -1.162596654$
    (d) $1.326196852 = y(0.54) \approx 1.294211496$, $1.661361751 = y(0.94) \approx 1.655028874$

15. (a) $h = 10^{-n/2}$
    (b) The minimal error is $10^{-n/2}(e-1) + 5e10^{-n-1}$.
    (c)

t	$w(h=0.1)$	$w(h=0.01)$	$y(t)$	Error ($n=8$)
0.5	0.40951	0.39499	0.39347	1.5×10^{-4}
1.0	0.65132	0.63397	0.63212	3.1×10^{-4}

16. We have

j	t_j	w_j
20	2	0.702938
40	4	-0.0457793
60	6	0.294870
80	8	0.341673
100	10	0.139432

17. (b) $w_{50} = 0.10430 \approx p(50)$
    (c) Since $p(t) = 1 - 0.99e^{-0.002t}$, $p(50) = 0.10421$.