1. The Composite Simpson's rule gives:
   (a) 0.5284163
   (b) 4.266654
   (c) 0.4329748
   (d) 0.8802210

2. The Composite Simpson's Rule gives:
   (a) 1.076163
   (b) 20.07458

3. The Composite Simpson's rule gives:
   (a) 0.4112649
   (b) 0.2440679
   (c) 0.05501681
   (d) 0.2903746

4. The Composite Simpson's Rule gives:
   (a) 1.1107218 with $n = 16$
   (b) 0.58904782 with $n = 12$

5. The escape velocity is approximately 6.9450 mi/s.

6. The polynomial $L_n(x)$ has n distinct zeros in $[0, \infty)$. Let $x_1, ..., x_n$ be the n distinct zeros of L_n and define, for each $i = 1, ..., n$,

$$c_{n,i} = \int_0^\infty e^{-x} \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}\, dx.$$

Let $P(x)$ be any polynomial of degree $n-1$ or less, and let $P_{n-1}(x)$ be the $(n-1)$th Lagrange polynomial for P on the nodes $x_1, ..., x_n$. As in the proof of Theorem 4.7,

$$\int_0^\infty P(x)e^{-x}\, dx = \int_0^\infty P_{n-1}(x)e^{-x}\, dx = \sum_{i=1}^{n} c_{n,i}P(x_i),$$

so the quadrature formula is exact for polynomials of degree $n-1$ or less.

If $P(x)$ has degree $2n-1$ or less, then $P(x)$ can be divided by the nth Laguerre polynomial $L_n(x)$ to obtain

$$P(x) = Q(x)L_n(x) + R(x),$$

where $Q(x)$ and $R(x)$ are both polynomials of degree less than n. As in proof of Theorem 4.7, the orthogonality of the Laguerre polynomials on $[0, \infty)$ implies that

$$Q(x) = \sum_{i=0}^{n-1} d_i L_i(x),$$

for some constants d_i.

Thus

$$\begin{aligned}\int_0^\infty e^{-x}P(x)\, dx &= \int_0^\infty \sum_{i=0}^{n-1} d_i L_i(x)L_n(x)e^{-x}\, dx + \int_0^\infty e^{-x}R(x)\, dx \\ &= \sum_{i=0}^{n-1} d_i \int_0^\infty L_i(x)L_n(x)e^{-x}\, dx + \sum_{i=1}^{n} c_{n,i}R(x_i) \\ &= 0 + \sum_{i=1}^{n} c_{n,i}R(x_i) = \sum_{i=1}^{n} c_{n,i}R(x_i).\end{aligned}$$

But

$$P(x_i) = Q(x_i)L_n(x_i) + R(x_i) = 0 + R(x_i) = R(x_i),$$

so

$$\int_0^\infty e^{-x}P(x)\, dx = \sum_{i=1}^{n} c_{n,i}P(x_i).$$

Hence the quadrature formula has degree of precision $2n-1$.

7. With $n = 2$ we have

$$\int_0^\infty e^{-x} f(x)\,dx \approx 0.8535534 f(0.5857864) + 0.1464466 f(3.4142136),$$

and with $n = 3$ we have

$$\int_0^\infty e^{-x} f(x)\,dx \approx 0.7110930 f(0.4157746) + 0.2785177 f(2.2942804). + 0.0103893 f(6.2899451)$$

8. For $n = 2$ we have 0.9238795. For $n = 3$ we have 0.9064405.

9. For $n = 2$ we have 2.9865139. For $n = 3$ we have 2.9958198.