1. Gaussian quadrature gives:

   (a) 0.1922687
   (b) 0.1594104
   (c) −0.1768190
   (d) 0.08926302
   (e) 2.5913247
   (f) −0.7307230
   (g) 0.6361966
   (h) 0.6423172

2. Gaussian quadrature with $n = 3$ gives:

   (a) 0.1922594
   (b) 0.1605954
   (c) −0.1768200
   (d) 0.08875385
   (e) 2.5892580
   (f) −0.7337990
   (g) 0.6362132
   (h) 0.6427011

3. Gaussian quadrature gives:

   (a) 0.1922594
   (b) 0.1606028
   (c) −0.1768200
   (d) 0.08875529
   (e) 2.5886327
   (f) −0.7339604
   (g) 0.6362133
   (h) 0.6426991

4. Gaussian quadrature with $n = 5$ gives:

   (a) 0.1922594
   (b) 0.1606028
   (c) −0.1768200
   (d) 0.08875528
   (e) 2.5886286
   (f) −0.7339687
   (g) 0.6362133
   (h) 0.6426991

5. $a = 1,\ b = 1,\ c = \frac{1}{3},\ d = -\frac{1}{3}$

6. $a = \frac{7}{15},\ b = \frac{16}{15},\ c = \frac{7}{15},\ d = \frac{1}{15},\ e = -\frac{1}{15}$

7. The Legendre polynomials $P_2(x)$ and $P_3(x)$ are given by

$$P_2(x) = \frac{1}{2}\left(3x^2 - 1\right) \quad \text{and} \quad P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right),$$

so their roots are easily verified.

For $n = 2$,

$$c_1 = \int_{-1}^{1} \frac{x + 0.5773502692}{1.1547005}\, dx = 1$$

and

$$c_2 = \int_{-1}^{1} \frac{x - 0.5773502692}{-1.1547005}\, dx = 1.$$

For $n = 3$,

$$c_1 = \int_{-1}^{1} \frac{x(x + 0.7745966692)}{1.2}\, dx = \frac{5}{9},$$

$$c_2 = \int_{-1}^{1} \frac{(x + 0.7745966692)(x - 0.7745966692)}{-0.6}\, dx = \frac{8}{9},$$

and

$$c_3 = \int_{-1}^{1} \frac{x(x - 0.7745966692)}{1.2}\, dx = \frac{5}{9}.$$

8. Let $P(x) = \prod_{i=1}^{n}(x - x_i)^2$. Then $Q(P) = 0$ and $\int_{-1}^{1} P(x)\, dx \neq 0$.

9. (a) The approximations and errors using Maple's routine with

$f := x^2 \cdot exp(x); a := -1; b := 1;$
$Quadrature(f(x), x = a..b, method = gaussian[n], partition = p, output = information)$

give the following approximations to the exact 10-digit value 0.878884623.

n	p	Number of Function Evaluations	Approximation	Error
8	1	8	0.878884623	0.0
4	2	8	0.878884546	8×10^{-8}
2	8	8	0.878387796	5×10^{-4}

(b) Gaussian quadrature chooses the evaluation points in an optimal way for the given interval. If the interval is partitioned it uses then uses points that are not optimal in the Gaussian sense, and less accuracy is to be expected.