

1. Simpson's rule gives

- (a) $S(1, 1.5) = 0.19224530$, $S(1, 1.25) = 0.039372434$, $S(1.25, 1.5) = 0.15288602$, and the actual value is 0.19225935.
- (b) $S(0, 1) = 0.16240168$, $S(0, 0.5) = 0.028861071$, $S(0.5, 1) = 0.13186140$, and the actual value is 0.16060279.
- (c) $S(0, 0.35) = -0.17682156$, $S(0, 0.175) = -0.087724382$, $S(0.175, 0.35) = -0.089095736$, and the actual value is -0.17682002 .
- (d) $S(0, \frac{\pi}{4}) = 0.087995669$, $S(0, \frac{\pi}{8}) = 0.0058315797$, $S(\frac{\pi}{8}, \frac{\pi}{4}) = 0.082877624$, and the actual value is 0.088755285.
- (e) $S(0, \frac{\pi}{4}) = 2.5836964$, $S(0, \frac{\pi}{8}) = 0.33088926$, $S(\frac{\pi}{8}, \frac{\pi}{4}) = 2.2568121$, and the actual value is 2.5886286.
- (f) $S(1, 1.6) = -0.73910533$, $S(1, 1.3) = -0.26141244$, $S(1.3, 1.6) = -0.47305351$, and the actual value is -0.73396917 .
- (g) $S(3, 3.5) = 0.63623873$, $S(3, 3.25) = 0.32567095$, $S(3.25, 3.5) = 0.31054412$, and the actual value is 0.63621334.
- (h) $S(0, \frac{\pi}{4}) = 0.64326905$, $S(0, \frac{\pi}{8}) = 0.37315002$, $S(\frac{\pi}{8}, \frac{\pi}{4}) = 0.26958270$, and the actual value is 0.64269908.

2. Adaptive quadrature gives:

- (a) 0.19226
- (b) 0.16072
- (c) -0.17682
- (d) 0.088709
- (e) 2.58770
- (f) -0.73447
- (g) 0.63622
- (h) 0.64273

3. Adaptive quadrature gives:

	Adaptive Quadrature Approximation	Actual Integral
(a)	2.00000103	2.00000000
(b)	1.37296499	1.372964103
(c)	0.23222233	0.23222150
(d)	5.11383291	5.113832671

4. Adaptive quadrature gives:

- (a) 108.555281
- (b) -1724.966983
- (c) -15.306308
- (d) -18.945949

5. Adaptive quadrature gives:

	Simpson's rule	Number evaluation	Error	Adaptive quadrature	Number evaluation	Error
(a)	-0.21515695	57	6.3×10^{-6}	-0.21515062	229	1.0×10^{-8}
(b)	0.95135226	83	9.6×10^{-6}	0.95134257	217	1.1×10^{-7}
(c)	-6.2831813	41	4.0×10^{-6}	-6.2831852	109	1.1×10^{-7}
(d)	5.8696024	27	2.6×10^{-6}	5.8696044	109	4.0×10^{-9}

6. Adaptive quadrature gives

$$\int_{0.1}^2 \sin \frac{1}{x} dx \approx 1.1454 \quad \text{and} \quad \int_{0.1}^2 \cos \frac{1}{x} dx \approx 0.67378.$$

7. $\int_0^{2\pi} u(t) dt \approx 0.00001$

8. (a) $c_1 = -\frac{1}{40}$, $c_2 = \frac{9}{680}$

(b) $\int_0^{2\pi} u(t) dt \approx -0.02348194$

9. We have, for $h = b - a$,

$$\left| T(a, b) - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right) \right| \approx \frac{h^3}{16} |f''(\mu)|$$

and

$$\left| \int_a^b f(x) dx - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right) \right| \approx \frac{h^3}{48} |f''(\mu)|.$$

So

$$\left| \int_a^b f(x) dx - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right) \right| \approx \frac{1}{3} \left| T(a, b) - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right) \right|.$$

10. For t between 0 and 1 we have the following values.

t	$c(t)$	$s(t)$
0.1	0.0999975	0.000523589
0.2	0.199921	0.00418759
0.3	0.299399	0.0141166
0.4	0.397475	0.0333568
0.5	0.492327	0.0647203
0.6	0.581061	0.110498
0.7	0.659650	0.172129
0.8	0.722844	0.249325
0.9	0.764972	0.339747
1.0	0.779880	0.438245